

Nature of Numbers

By

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Introduction

मङ्गलाचरण

I derive my authority to write this book by command of *Jagadguru Śrī 1008 Swāmī Nīśchalānanda Saraswatī*, head of *Govardhana Pīṭha, Puri* in tradition starting with *Śrī Nārāyaṇa*. He was followed by *Brahmā* born from lotus, *Vasiṣṭha, Śakti*, his son *Parāśara, Vyāsa, Śuka*, the great *Gauḍpāda, yogi Govindapāda* and his disciple *Śaṅkarāchārya* whose 4 disciples continued the tradition at 4 *pīṭhas-Padmapāda, Hastāmalaka, Toṭaka, and Sureśvara*, author of *Vārttika* (short notes on *Upaniṣads*). That tradition continues till our present *guru*.

नारायणं पद्मभवं वसिष्ठं शक्तिञ्च तत्पुत्र पराशरञ्च

व्यासं शुक्रं गौड़पदं महान्तं गोविन्द योगीन्द्रमथास्यशिष्यम् ।

श्रीशङ्कराचार्यमथास्यपद्मपादञ्च हस्तामलकञ्च शिष्यम्

तं तोटकं वार्तिककारमन्यान्स्मद्गुरुन् सन्ततमानतोऽस्मि ॥

The same tradition from *Nārāyaṇa* to *Swāmī Nīśchalānanda* with *Śaṅkāraachārya* in middle is praised below.

नारायण समारम्भां शङ्कराचार्य मध्यमाम् ।

निश्चलानन्द पर्यन्तां वन्दे गुरुपरम्पराम् ॥

There is another start of the tradition from *Sadāśiva* to 4 mental sons of *Brahmā-Sanak, Sanadana, Sanātana, Sanatkumāra*. Their disciple was *Nārada* who gave the knowledge to *Vyāsa* and his son *Śuka*. From them the tradition is same.

सदाशिव परम्परा-

सदाशिवं श्रीसनकं सनन्दनं सनातनञ्चैव सनत्कुमारम्

श्री नारदं व्यासशुक्रौ च गौड़पादञ्च गोविन्दपदं महान्तम् ।

श्री शङ्कराचार्यमथास्य पद्मपादञ्च हस्तामलकञ्च शिष्यम्

तन्तोटकं वार्तिककारमन्यान्स्मद्गुरुन् सन्ततमानतोऽस्मि ॥

Respected *swamī Nīśchalānanda jī* is also direct disciple of *Swāmī Hariharananda jī*, famous as *Karapātrī jī* who spent his whole life in re-establishing Vedic path by his learned commentaries on *vedas, Rāmāyaṇa, Bhāgavata* etc. and political and social struggles.

केचिद्भजन्ति विबुधा हरिमीशितार-

ञ्चान्ये हरं भवनिदाघहरं श्रयन्ति ।

धर्माऽऽर्तिखिन्नमनसां किमुपासनीयं

प्रत्यक्षितं हरिहराद्वयमद्वयन्ः ॥

Myself- *Arun Kumar Upadhyay* of *Bahradvāja gotra* was guided into devotion to the tradition of sages by my parents, which is sole basis of my knowledge. To quote *Swāmī Karapātrī jī* from his *Vedārtha-Pārijāta* (1/32)

येषां पितृपितामहादिपुरुषा आसन् सदा वैदिका

ये स्वान्ते परिशीलयन्ति सततं वेदान् सदर्थान्वितान् ।

वेदद्विङ्भिर्हृदीरितानभिनवानर्थानुदीक्ष्य स्वयं

ये क्लिश्यन्ति महत्तदर्थमखिलो ह्यस्माकमेष श्रमः ॥

माता जगत्तारिणी देवी पिता श्री चन्द्रशेखरः ।

संस्काराणि तयोर्दत्ताः क्षेमाय प्रणमाम्यहम् ।

2. Books on Mathematics by *Swāmī jī* Earlier 143rd head of *Govardhana Pīṭha*, *Swāmī Bharatikṛṣṇa Tīrtha jī* (1884-1960) had written Vedic Mathematics (1978) and Vedic Metaphysics-both published by M/s. Motilal Banarsidass, Delhi. Vedic Mathematics explains 16 *gaṇita-sūtras* from an appendix of *Rgveda*. This was intended to be first volume only to explain short methods of complicated calculation. The other volumes were about higher mathematics which were probably stolen and are untraced till now. Some more applications of faster numerical methods were published by others-e.g. the following books by Motilal Banarsidas-Vedic Mathematics Decades : Space Book, Astronomical Application of Vedic Mathematics, Vedic Mathematics for Schools (3 parts), Vertically and Cross-wise-Applications of Vedic Mathematics *Sūtras*. Prof Kenneth Williams of Glasgow, UK has dedicated his life to write many books for popularising these methods at various levels. In India, Dr. Narinder Puri of Roorkee, Sri Viśvanath Unkolkar of Dandeli (Karnataka) etc. have written major books in the field.

Continuing the mathematical tradition of *Govardhana Pīṭha*, *Swāmī Nīśchalānanda Saraswatī jī* has written 3 books on philosophy of mathematics-specially on origin and nature of numbers-

1. *Swastika Gaṇita*-This describes indications of numbers by *Āryabhaṭa*, *Swastika* sign, decimal system, significance of 0,1,108, signs of addition, subtraction, multiplication, division, concept of zero etc.

2. *Gaṇita-Darśana*-This gives concept of zero as *Puruṣa* (cosmic man), as a number.

3. *Aṅka-Padīya (Gaṇanātha-Gaṇita)*-Content of zero and development of numbers from that.

The subject matter of 3 books is different, but has a lot of common elements to explain the context, so all the three have been summarized in one book. It is difficult to select a suitable title of the book to indicate the subject matter. There are several books on philosophy of modern and ancient mathematics. Subject of these books is unique and untouched, so it was necessary to give a distinct title. For months, I was thinking about a proper title, and still it is not fully proper, but some name has to be given. There is no literature like this, and it was difficult to grasp the thoughts. Though *Swāmī jī* was kind to trust me with this work, the delay in understanding the matter created impression of my negligence. The problem is also to make normal educated people understand. Persons having read modern mathematics will need a comparison with modern theories of zero and infinity and give examples of similar views in vedic literature. Similarly, persons having studied sanskrit or vedic texts have to be explained mathematical implications which has been done by *Swāmī jī* in brief manner with minimum number of quotes. Only one or two extra verses have been quoted, as they are exact parallels to modern definitions. There are still many other passages, but only a sample is given to explain the points. Some of the ideas were not clear to me and given in short. Notes by me include-Cantor Set theory about grades of infinity, Vīrasena classes of Infinity in Jain text Dhavalā, a verse of Kathopaniṣad giving exact definition of zero limit and infinity, some new meanings of numeral word e.g. cloud for 17.

Chapter 1

Zero and Infinity

1. Definition of zero-Normal meaning of zero is that when a number is subtracted from itself, nothing remains. That nothing is called zero. In symbols-

$$a-a=0, \text{ or } 2-2=0 \text{ etc.}$$

However, zero is not exactly nothing. It has two functions in mathematics-

(1) It is start of counting, hence it is also a number. Not only as a start, but it occurs after every 10 numbers-e.g. 10, 20, 30, etc.

(2) Another meaning is that it is very small number, rather smaller than the smallest. That concept is used in analysis of numbers-real or complex. In a fraction, if we increase the value of denominator, value of function decreases. The fraction tends to zero when denominator tends to infinity. Here, another concept is needed-infinity. Infinity is defined to be larger than any number we can imagine. This is reverse of zero. When the denominator of a fraction decreases indefinitely, its value tends to infinity.

To understand the symbolic mathematics, we have to know about relation between two variables, called a function-

$f(x)$ is called function of x . Here, x is independent variable. It can take any value. Any unknown quantity in algebra is taken as x and we build an equation based on its description. By solving that equation, we find the value of x . In this method, x is seed (*bīja*) from which the tree of equation is built. So the method is called *Bīja-gaṇita*. As x takes arbitrary value, it is

called *Gulika* or *ka* in short. A quantity dependent on x is called function of x , e.g. $2x$ is a function of x called $f(x)$. When x is 1, $f(x)=2x$ is 2, when $x=2$, $f(x)=4$ and so on. Thus $f(x)$ is dependent variable. In sanskrit, it is called *yāvat-tāvat* i.e. it is as much (*tāvat*) based on x (*yāvat*). In short, it is called *ya*. *Ka* and *ya* of sanskrit *Bījagaṇita* have become x and y in modern algebra. Algebra word itself is a translation of *Brahma*. *Brahmagupta* had written his famous astronomy book-*Brāhma-sphuṭa-siddhānta* in 622 AD which contained mathematical methods like algebra, trigonometry also. This was translated in 632 AD under orders of *Khalifa Mansur* in Arabic. The name *Brāhma* (=of *Brahma*) was translated as *Al-Zabar* (the Great or the Powerful) which is meaning of *Brahma* in sanskrit based on verb-*Br̥ṇha*=to grow. *Siddhānta* (*sphuṭa*) was translated as *muquābalā* (contest or comparison-because correctness is decided by *śāstrārtha*=contest). The complete book on mathematics was called *Al-zabar-ul-muquābalā*. When it was translated into Latin later on, only the first part was retained-*Al-zabar* which has become algebra. *Bhaviṣya purāṇa* tells that Mohammed had requested help from *Bhoja* king of *Mālvā* for help in his holy war. Islamic tradition also admits help by Indians, in particular, the brahmins who helped Mohammed were called *Mohayālī* (helping Mohammed). Thus, one of the purposes of the book by *Brahmagupta* may be to give basis of new Islamic calendar starting in 622 AD. Plea of Mohammed was that he wanted to re-establish rule of *Dharma* as it existed in Arab when it was under rule of *Vikramāditya*, fore-father of *Bhoja*. Though, Arabs sought guidance of India in war and mathematics, there

were persons in Arab who knew Arabic, sanskrit and mathematics who did the translation and read it. The translation of *Brahma* as Algebra is not incidental-the concepts of zero and infinity-both indicate features of *Brahma*. Now coming to point, zero is defined as

$$f(x)=1/x$$

$$f(x) \rightarrow 0 \text{ when } x \rightarrow \infty$$

Here, ∞ =infinity, $f(x)$ is called function of x .

\rightarrow means tends to.

For example, $1/2 = 0.5$

$$1/3=0.333..., 1/4=0.25, 1/5=0.2, 1/6=0.1666...$$

Thus, we see that in $1/x$, when we increase x in sequence 2, 3, 4, 5, 6 ..., value of $1/x$ declines steadily. We can make it smaller than any small number by taking x sufficiently large. Suppose we take a small number as 1 crore part of 1 written as 10^{-7} , then x will be taken as bigger than 10^7 . Any imaginary small number is called ϵ (Greek letter epsilon).

Thus, $1/x < \epsilon$, when $x > G$ (any large number)

Here, $<$ = smaller than, $>$ =bigger than.

$x > G$ (x is bigger than G) means same as $x \rightarrow \infty$ (x tends to infinity).

2. Indian concept-The above definitions of 0 and ∞ are same as the concept of God described in *Kaṭha-upaniṣad* (2/20) -also in *Nārada-parivṛājaka-upa.*(9/13), *Śarabha-upa.*(19), *Nārāyaṇa upa.*(1/3)-

अणोरणीयान्महतो महीयान्, आत्मास्य जन्तोर्निहितो गुहायाम् ।

तमक्रतुःपश्यति वीतशोको धातु प्रसादान्महिमानमात्मनः ॥

I.e. the *ātmā* living in heart of this being is smaller than the smallest and larger (or greater) than the greatest. Only the self-less person blessed by God can see His greatness. There are many similar lines in *upaniṣads* and other vedic texts given without translation-

अणिष्ठो वाङ्मेऽङ्गे समानयति (मैत्रायणी उप. २/६)

अणीयान् ह्यतर्क्यमणुप्रमाणात् । (कठोपनिषद् १/२/८)

अणीयान् ब्रीहेर्वा यवाद्वा । (छान्दोग्य उप. ३/१४/३)

अणु कोटर विस्तीर्णे त्रैलोक्यं च जगद्भवेत् । (तेजबिन्दु उप. ६/८७)

अणोरणीयानहमेव तद्वत् । (कैवल्य उप. २०)

अणोरणीयंसमनुस्मरेद्यं. (गीता ८/९)

अणोरप्यण्व्यं ध्यात्वा.. (मैत्रायणी उप. ६/३८)

Sanskrit word for zero is *śūnya* meaning void or empty. Its Arabic translation is *sifr* which became Zephirum in Latin. This was called *cifra* by Leonardo Fibonacci (1170-1250 AD). in his book on Hindu place value systems called Liber Abacci. This became *chiffre* in French, *ziffer* in German, *Zefiro* in Italian and *zero* in English.

Līlāvati by *Bhāskarāchārya-II* (B.1114 AD) gives rules for zero-

योगे खं क्षेपसमं वर्गादौ खं भाजितो राशिः ।

खहरः स्यात्खगुणः खं खगुणश्चिन्त्यश्च शेषविधौ ॥

शून्ये गुणके जाते खं हारश्चेत् पुनस्तदा राशिः ।

अविकृत एव ज्ञेयस्तथैव खेनोनितश्च युतः ॥

In addition, cipher makes the sum equal to the additive. In involution (square and cube of zero) and evolution (i.e. square-root and cube-root of zero), the result is zero. A definite quan-

tity, divided by cipher, is a fraction with its denominator cipher (ख-छेद). The product of cipher is naught, but it must be retained as a multiple of cipher, if any further operation impend. Cipher having become a multiplier. If cipher becomes a divisor, the definite quantity must be understood to be unchanged. So likewise any quantity, to which cipher is added, or from which it is subtracted, (is unaltered). Symbolically-
 $a-a=0$, $a \pm 0 = a$, $0 \pm a = \pm a$, $0 \times (\pm a)=0$, $0 \times 0=0$, $0/a=0$.

Concept of limit has been explained in *Bīja-gaṇita* of *Bhāskara-II* while explaining the strange property of a fraction with zero denominator-

अस्मिन् विकारः खहरे न राशावपि प्रविष्टेष्वपि निःसृतेषु ।
 बहुष्वपि स्याल्लयसृष्टिकालेऽनन्तेऽच्युते भूतगणेषु यद्वत् ॥

A fraction having zero denominator has the strange property similar to that of Almighty. The Almighty absorbs infinite beings of the universe during the deluge and lets out infinite beings during the creation, but he does not undergo any change. Similarly, the value of a fraction with denominator zero does not undergo any change by the addition of any huge number to it or subtraction of any huge number from it.

Symbolically-

Value of $(n/0) + \text{any large number} = \text{Value of } (n/0)$

Value of $(n/0) - \text{any large number} = \text{Value of } (n/0)$

Concept of limit and infinitesimal calculus is also given in *Siddhānta-śiromaṇi* of *Bhāskara-II*-

बिम्बार्धस्य कोटिज्यागुणस्त्रिज्याहरः फलं दोज्योयोरन्तरम् ।

This means, $(\sin x - \sin x') = (x - x') \cos x$

This is equivalent to, $d(\sin x) = (\cos x)dx$ as in modern differen-

tial calculus. In common language, small change in value of $\sin x$ is proportional to change in x multiplied by $\cos x$.

3. Zero as non-entity like *Puruṣa*-*Puruṣa* is the eternal conscious being without any change and present all over all the time. Due to lack of change, it is beyond perception, so it is called non-element. Forms of nature are perceived as they suffer change by action of 3 *guṇas*. The 3 *guṇas* are seen in two ways. Firstly, they are 3 distinct modes of existence. Due to their combination, *prakṛti* (nature) exists in $2^3 = 8$ forms. In each form a *guṇa* like *sattva* may exist or not, giving 2 varieties. Second *guṇa*, *rajas* also can exist or not-total combinations joined with the first *guṇa* are $2 \times 2 = 4$. Similarly, the third *guṇa*-*taṃas* also will multiply the forms to $4 \times 2 = 8$. These 8 varieties of *prakṛti* are *prakṛti* (creative) and *vikṛti* (transformed or created) both. The root form or formless *prakṛti* is *mūla-prakṛti*. In 8 forms of *prakṛti*, all 3 *guṇa* are similar. In their subsequent transformation, the *guṇa* are different. *Sattva* is potential or energy of action, *rajas* is actual action or change. *Tamas* is inactive. Thus 8 *prakṛti* give rise to $8+8=16$ *vikṛti*, which is only created version, it can not create further. 8 *Prakṛti* + 16 *vikṛti* make a total of 24 *prakṛti* which are modes of creation due to 3 *guṇas*. *Puruṣa* remains aloof from the changes and creation, thus being the 25th element of *Sāṅkhya* philosophy (*saṅkhyā*=number, *Sāṅkhya* = number of classes). Being without change and aloof, it is non-element also. This is expressed in *Mahābhārata*, *śānti-parva* chapter 305 as-

अनीश्वरमतत्त्वं च तत्त्वम् (४१)

I.e. the (25th) *tattva* (element) is not cause or effect and hence

non-*tattva*. *Īśvara* controls everybody, being aloof from change is lack of that power or *anīśvara*.

निस्तत्त्वः पञ्चविंशकः (४३)

The 25th element is non-*tattva*, or not connected with other *tattvas*. The complete verses are-

अव्यक्तं क्षेत्रमित्युक्तं तथा सत्त्वं तथेश्वरः ।

अनीश्वरमतत्त्वं च तत्त्वं तत् पञ्चविंशकम् ॥४१॥

तत्त्वानि च चतुर्विंशत् परिसङ्ख्याय तत्त्वतः ।

सङ्ख्याः सह प्रकृत्या तु निस्तत्त्वः पञ्चविंशकः ॥४३॥

(महाभारत, शान्तिपर्व, अध्याय ३०५)

There are many other quotes-e.g.

मायां तु प्रकृतिं विद्यान्मायिनं तु महेश्वरम् ।(श्वेताश्वरोपनिषद् ४/१०)

Understand *māyā* as *prakṛti* (nature) and its lord (*Mayī*) as *Maheśvara*.

प्रकृतिर्माया पुरुषः शिवः ।(गणेशपूर्वतापिनीयोपनिषद् २/३)

Prakṛti is *māyā* and *Puruṣa* is *Śiva*.

अविद्यामूलप्रकृतिर्माया लोहितशुक्लकृष्णा ।(शाण्डिल्योपनिषद् ३/१)

Avidyā is root *Prakṛti* or *māyā*, which is 3-fold-red, white and black (3 names for 3 *guṇa-sattva, rajas, tamas*)

ब्रह्मशक्तिरेव प्रकृतिः ।(निरालम्बोपनिषद्)

Prakṛti is nothing but power of *Brahma*.

चैतन्यस्यैकरूपत्वाद्भेदो युक्तो न कर्हिचित् ।(योगशिखोपनिषद् ४/१)

The Conscious Being is of 1 form only, so it can not have any parts.

सत्यं ज्ञानमानन्दं ब्रह्म ।(सर्वसारोपनिषद् ३)

Brahma (the Supreme) is *satya* (truth, existence), *jñāna* (knowledge) and *ānanda* (bliss).

महाभूतोत्थसूक्ष्माङ्गोपाधिकाः सर्वे जीवाः ।

(त्रिपाद्विभूतिमहानारायणोपनिषद् १)

All beings have small organs arising from that Great Being. *Puruṣa* is beyond peception as it all-pervading and does not change. In that sense, it is non-entity like zero.

4. Dual role of zero-We start the counting in 2 ways-from zero as a number or from 1. For example, in petrol pumps, meter states that before start of pump, number is to be set at zero. The rulership of kings is counted from *Bhādra-sukla* 12th, when *Vāmana Viṣṇu* had started supremacy of *Devas* under *Indra*. Since, counting is from this date, it is called *Śūnya* (=zero). There is difference in number of years of kings, as there are two systems of counting. In one system, all numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11,.. are counted. In another system, numbers ending with 0 or 6 are not counted. The counting will be 1, 2, 3, 4, 5, 7, 8, 9, 11, 12,... etc. Thus in 35 years of rule there will be 43 numbers (called *Aṅka*) as 8 numbers will be omitted- 6, 10, 16, 20, 26, 30, 36, 40 and the counting will go up to 35 + 8 = 43. Thus the start of counting is from zero, but zero itself is not counted. Natural numbers used for counting are from 1 only. In theorems on Prime numbers, we exclude 1 also and take numbers from 2 as inclusion of 1 does not fit into theorems, Zero does not have any value of its own. But combined with a number, it contributes to the value. For example, 0 has no measure or value, but if it is kept after 1 or any other number, value of number increases 10 times. Value of 1 is 10-

1=9 more. By keeping two zeros at end, number becomes 100 times. I.e. 1 increases by $100-1=99$. Keeping zero in middle also increases the value. E.g. $102=12=90$, thus inserting zero in middle, increases value of 12 by 90. Increase in value of number by putting zero in that has been called *Meyāṅka*. *Meyāṅka* is the complete number which includes zero at some places. If zero is only at unit place, value of the remaining digits increases by 10. Thus it increases in ratio of 9 ($10-1$).

$250=25 \times 10$. Increase in value is $250-25=225=25 \times 9$.

If zero is at 10th place or both at unit and 10 places, then the number left to two places increases 99 ($100-1$) times.

$505-(5+5)=495=5 \times 99$

$500-5=495=5 \times 99$

Value of zero is actually value of the place in number. We are using only 10 digits including zero to write all numbers-0, 1, 2, 3, 4, 5, 6, 7, 8, 9. To write bigger number than 9, we use two places-one place is added to the left of first place where value of any number is 10 times. By keeping 1, 2, 3,.. etc. at this place we mean 10, 20, 30, .. etc. If no value is needed at unit or the first place, 0 (zero) is kept there. By using two places we can write $10 \times 10=100$ numbers including zero, i.e. upto 99. For writing bigger numbers, we use third place to the left having value of $10^2=100$. Three places can indicate $10^3=1000$ numbers, including zero i.e. upto 999. Similarly, we can write any number by using successive places to the left. So it is said that, digits (0,1.. to 9) have motion to the left direction-
अङ्कानां वामतो गतिः ।

(Place value of) digits (of a number) move to left (in multiples

of ten)

स्थानात् स्थानं दशगुणमेकस्माद् गुण्यते द्विज ।

ततोऽष्टादशमे भागे परार्द्धमभिधीयते । (विष्णुपुराण ६/३/४)

O *brāhmaṇa* ! from one place to the next one, the places are multiples of ten. At 18th place from 1 i.e. 10^{17} is *parārdha*.

यथा एकरेखा शतस्थाने दशस्थाने दशैवं चैकस्थाने यथा च एकत्वेऽपि स्त्री माता च उच्यते दुहिता स्वसा च इति । (योगसूत्र, व्यास भाष्य)

A woman is called mother (by her children), daughter (by her parents), daughter-in-law (by her parents-in-law). Similarly, the same digit assumes hundredth value, tenth value and unit value when it occupies hundredth place, tenth place and unit place.

For fractional numbers in this system, the place values are kept to the right of unit place and are decreasing successively in ratio of 10. Fraction part is separated by giving a dot after unit place.

Number Place values

	<u>Hundreds</u>	<u>Tens</u>	<u>Unit</u>	<u>1/10</u>	<u>1/100</u>	<u>1/1000</u>
234.542	2	3	4	5	4	2
Value of digits-200	30	4	5/10	4/100	2/1000	
567.12	5	6	7	1	2	0(no number)

Thus, $567.12=567.120$.

5. Zero also is existant-There are many sayings in philosophy texts indicating that words meaning zero like *asat* (intangible, non-existent, false), *abhāva* (lack, absence of matter), *nistattva* (non-element)-do not mean complete non-existence. They too indicate something which is not visible or percepti-

ble to us.

न भावाज्जायते भावो भावोऽभावान्न जायते ।

नाभावाज्जातेऽभावोऽभावो भावान्न जायते ॥ (बौद्धधर्म दर्शन पृ.५४८)

Bhāva (=abstract existence) can not come out of *bhāva* or even from *abhāva*. *Abhāva* also can not come from *bhāva* or from *abhāva*.

नासतो विद्यते भावो नाभावो विद्यते सतः ।

उभयोरपि दृष्टोऽन्तस्त्वनयोस्तत्त्वदर्शिभिः ॥ (भगवद्गीता, २/१६)

From *asat* can not come *bhāva* (existence) and from *sat* can not become *abhāva* (vanish). This is evident only to the seekers of truth.

नास्त्यसद्वेतुकमत्सदसद्वेतुकं तथा ।

सच्च सद्वेतुकं नास्ति सद्वेतुकमसत्कुतः ॥ (माण्डूक्यकारिका ४/४०)

Asat is not due to *asat* nor *sat* is from *asat*. Similarly, *sat* is not the reason of *sat* or *asat*.

निस्तत्त्वा कार्यगम्याऽस्य शक्तिर्मायाग्नि शक्तिवत् ।

न हि शक्तिः क्वचित्कैश्चिद्बुध्यते कार्यतः पुरा ॥४७॥

(पञ्चदशी, अध्याय २)

The difficulty in separating cause and effect makes *Brahma* as unique. *Śakti* is not perceived before some work is done by it.

In *Anka-Padiyam*, 13 points have been given to prove that zero means existence-

(1) Numbers from 1 to 9 indicate existence. Their association with zero increases their value. So zero also should mean existence.

(2) 1,2,.. etc. become 10,20,.. etc after being attached with

zero has no value then 10+1 will not be 11 but 1+1=2. This is successor of 9 in numbers as-

$$9+1=10, 99+1=100, 999+1=1000.$$

Non-existence can not lead to existence and vice-versa-
नासतो विद्यते भावो नाभावो विद्यते सतः (गीता २/१६)

(5) As it is impossible to count upto infinity, similarly only zero can not be counted. So it is not considered a number. Due to difficulty in visualising zero, common logic takes it as nill-

$$\frac{1}{0} = \infty$$

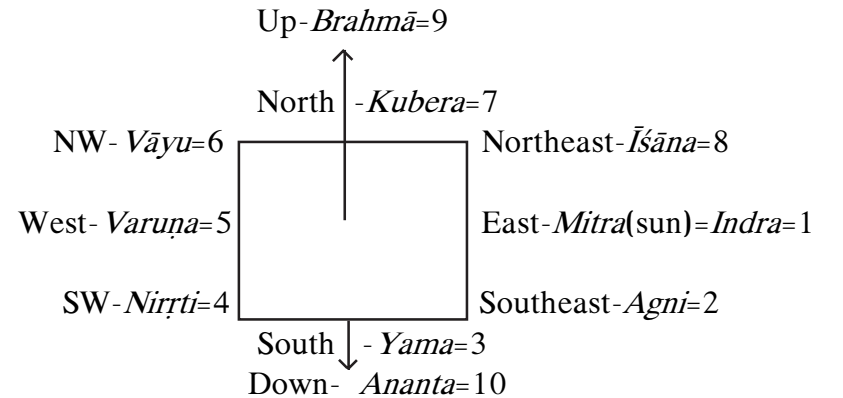
Divisor or remainder zero is not nill as it starts the sequence of numbers starting with 1.

कथमसतः सज्जायेत (छान्दोग्योपनिषत् ६/२/२) ।

After complete dissolution of world, the remaining abstract world is called un-differentiated remainder. Similarly, end of numbers is called zero.

शिष्यते शेष संज्ञकः ॥ (श्रीमद्भागवत १०/३/२५)

(6) The ultimate creator is abstract and formless-that is zero. Its forms with specific attributes like *Brahma* etc. are indicated by numbers from 1 to 10.



zero. Thus 1 zero after the number makes its value 10 times.

(3) Non-existence can not be counted as a number. But 10, 100, 1000 are counted as numbers.

(4) There is clear difference between $1+1=2$ and $10+1=11$. The ten directions and numbers are also indicated by ten *mahāvidyās*. *Dhūmavātī*, *Chhinnamastā*, *Tripurasundarī*, *Bagalā*, *Mātangi*, *Bhuvaneśvarī*, *Ṣoḍaśī*, *Tārā*, *Kamalā*-indicating numbers from 1 to 10.

0 and 1 both indicate *Brahma*. The supreme without any visible qualities or attributes is 0. As addition of 0 does not make any change, this aspect of *Brahma* is detached, without form or actions. *Brahma* as visible entities is indicated by 1 from which counting starts. By successive addition of 1, all numbers are formed. The 2 forms of *Brahma* have been called 2 birds in *Vedas* (*Brahma* or *ātmā* and *Jīva*) or Adam and Eve in Bible-
द्वा सुपर्णा सयुजा सखाया समानं वृक्षं परिषस्वजाते ।
तयोरन्यः पिप्पलं स्वाद्वत्त्यनश्नन्नन्यो अभिचाकशीति ।

(ऋग्वेद १/१६४/२०)

2 birds (*Ātmā* and *Jīva*) reside on the same tree (chain of creation) jointly and with friendship. Among the 2 neighbours, one is eating the *pippala* (fruit of acition=apple) and the other is detached and just keeps watch.

Chain of creation is parallel to sequence of numbers-0=*Ātmā*, which is also infinity, written as chain of 0s-∞ is beyond comprehension as to how un-attached formless *Brahma* became Creator with attributes and started creation with sky etc.-

को ददर्श प्रथमं जायमानमस्थन्वन्तं यदनस्था बिभर्ति ।

भूम्या असुरसृगात्मा क्व स्वित्को विद्वांसमुपगात्प्रष्टुमेतत् ।

(ऋग्वेद १/१६४/४)

Here, *Kah* =who, or *Karttā* (=Creator, Doer). Thus the verse is a question and answer also. Who saw the First creation ? Or, the Creator only saw the first creation. How the boneless gave rise to beings with bones ? Where were the *prāṇa* (life-force), blood and *ātmā* (conscious) of *bhūmī* (earth, or a closed structure). Who went to the knower to ask these ?

Numbers from 2 to 10 are core of its body (*sūkśma-śarīra*), 1 is the causal body (*kāraṇa-śarīra*) and 0 is its *ātmā*. Place of creation (*yajña-vedī* =place of *yajña*) earth is 0. 1 is navel of numbers as other numbers are created by successive addition of 1. Sun is *Vṛṣa* (=which rains or radiates, bull). It moves through 7 *aśva* (horse, driving force). Thus 2 to 8 are 7 horses of sun, 8 being the sun. 9 is *soma* (=moon, dispersed energy) after which numbers proceed upto infinity with 0. Thus 1,9, 0 are 3 navels of number system as *Aum* (ॐ)=A+U+M is of letters indicating *Brahmā*, *Viṣṇu* and *Maheśa*-
इयं वेदिः परो अन्तः पृथिव्या अयं यज्ञो भुवनस्य नाभिः ।
अयं सोमो वृष्णो अश्वस्य रेतो ब्रह्मायं वावः परमं व्योम ॥

(ऋग्वेद १/१६४/३५)

The number system is based on three navels-1,9,0, and 7 join in between-2, 3, 4, 5, 6, 7, 8. All pervading is the single zero-सप्त युज्जन्ति रथमेकचक्रमेको अश्वो वहति सप्तनामा ।
त्रिनामिचक्रमजरमनर्व यत्रेमा विश्वा भुवनानि तस्थुः ।

(ऋग्वेद १/१६४/२)

(7) Zero is not imaginary like castles in air. In multiplications,

zero indicates growth when used with other numbers, e.g.-

$$10 \times 10 = 100, 10 \times 50 = 500$$

But, alone in multiplication, it nullifies all-

$$1 \times 0 = 0, 2 \times 0 = 0.$$

(8) Non-existent can not be counted as a number nor it is mere distinction. The difference between 1 and 10 is of zero only and 10 is a number like 1.

(9) Deduction of non-existent means existence. But $0-0=0$. Hence zero is not non-existent.

(10) Zero is contained in all numbers. By adding 1, 2, 3, 4, 5, 6, 7, 8, 9 to 10 we get 11, 12, 13, 14, 15, 16, 17, 18, 19. We get the numbers in place of zero. Similarly $0+0=0$.

(11) Non-existence is not minimum state of existence, nor existence is developed state of non-existence. 0 and 1 come in same sequence. $0+1=1$ and $1-1=0$, thus 0 is just less than 1 or 1 is more than 0.

(12) *Mīmāṃsā*-As per this thought, change of *Bhāva* (existence) is called *Abhāva* (lack of *bhāva*). One mode of existence is lack of it as per the other mode-

भावान्तरभावो हि कयाचित्तु व्यपेक्षया (श्लोकवार्तिक, पृष्ठ ५६६).

(12-2) Change of place also makes an object non-existent at the original place.

(12-3) Change of time also makes the object non-existent at the original time. Destruction of an object is really change of its form. No object is finished in all components. No mis-giving (*bhrama*) is base-less. No obstruction (*bādha*) is beyond time limit-

न हि निरधिष्ठानो भ्रमोऽस्ति । न वा निरवधिर्बाधः । (श्रीमद्भागवत,

मधुसूदनी टीका १/१/१)

Brahma-purāṇa also tells that *Maharloka* becomes zero after *kalpa* (creation) but is not destroyed. This means that zero also has some existence-

शून्यो भवति कल्पान्ते योऽन्तं न च (/यो ह्यन्ते न) विनश्यति ।

(ब्रह्म पुराण २३/२०)

Collection of zeros is infinite (*ananta*), immeasurable (*aprameya*), and innumerable (*asankhyāta*).

अनन्तस्य न तस्यान्तः सङ्ख्यानं चापि विद्यते ।

तदनन्तमसङ्ख्यातं प्रमाणेनापि वै यतः । (ब्रह्म पुराण २३/२६)

The Supreme *Devī* in female form (Eternal field) has called herself as zero and non-zero. She is zero witness to the zeros-
शून्यं चाशून्यं च (देव्यथर्वशीर्ष, २)

शून्यानां शून्यसाक्षिणी (देव्यथर्वशीर्ष, २४)

(13) *Puruṣa* is zero as it is beyond perception, not involved in work, empty field (called *Devī* in female form), neither material nor matter etc. It is one as it is same essence of all varieties. After mixing with creative element (*Prakṛti*), it is visible as one essence. When it is called one, it means there is an alternative. many alternative forms give rise to further numbers-1, 2, 3, 4,... etc.

अव्यक्तं क्षेत्रमित्युक्तं तथा सत्त्वं तथेश्वरः । अनीश्वरमतत्त्वं तत् पञ्चविंशकम् ।।
तत्त्वानि च चतुर्विंशत् परिसंख्याय तत्त्वतः । संख्याः सह प्रकृत्या तु निस्तत्त्वः
पञ्चविंशकः । (महाभारत, शान्तिपर्व, ३०६/४१, ४३)

6. Definition of infinity-To understand infinity, it is useful to know types of numbers in modern mathematics.

The numbers

...., -3, -2, -1, 0, 1, 2, 3,

are called rational integers, or simply integers.

The numbers 0, 1, 2, 3,are the *non-positive integers*.

The numbers 1, 2, 3, 4, are *positive integers*.

These are also called *natural numbers* or *cardinal numbers* used in counting.

Any number p is a prime if-

(i) $p > 1$.

(ii) p has no positive divisors except 1 and p .

The first primes are-

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 41, 43, 53,

Rational number can be written as a fraction ($\pm p/q$), where p and q are positive integers. It is either written as a fraction p/q , meaning p is divided by q , or represented by a decimal.

$$\begin{array}{l} \alpha_0.\alpha_1\alpha_2.... \\ \text{q) } \underline{\quad\quad\quad} \\ \beta_0 = 10 \times (p - q \times \alpha_0) \\ \beta_1 = 10 \times (\beta_0 - q \times \alpha_1) \\ \dots\dots \end{array}$$

where α_0 is a non-negative integer and α_k ($k=1, 2, \dots$) are decimal digits. If p and q are each multiplied by same number, the decimal remains the same, e.g.

$$4/10 = 2/5 = 0.4.$$

Decimal numbers terminate if the divisor has factors of 2 and 5 only, as $10 = 2 \times 5$. Otherwise, if the divisor has other factors (e.g. $15 = 3 \times 5$, 7 etc), it does not terminate. Suppose, a number is divided by 7 for decimal representation. Then the remainder

has to be less than 6. It will not be zero, as the process will stop. Thus, remainders will be 1, 2, 3, 4, 5, 6 only. After 7th step, one of the remainders will be repeated, thereafter the same numbers will come. Thus, the decimal will recur maximum after 6 terms. To convert the decimal number to fraction, we put 1 in denominator (divisor) and as many zeros as the non-recurring digits and 9 for each recurring digit.

Thus, $0.4 = 4/10$ or $2/5$.

$103/330 = 0.3121212\dots = 0.3(12)$ -here 12 recur or come repeatedly. To convert it back to fraction

$$0.3(12) = \frac{3}{10} + \frac{12}{990} = \frac{103}{330}$$

The numbers which can not be written as fraction are called Irrational numbers. These are of two types-

(a) Algebraic irrationals- are irrationals. They can not be written as fraction but are solutions of algebraic equations. e.g.

$$e^{\sqrt{2}}, e^{\sqrt{5}}, \sqrt{7}e^{3\sqrt{2}}, \log 2$$

$\sqrt{2}$ is solution of equation $x^2=2$.

(b) The numbers e and π and numbers derived from them-It has been proved that the following numbers are irrational-

$$\pi = \frac{\text{Circumference}}{\text{Diameter}}$$

$$e = 1 + \frac{1}{1} + \frac{1}{1.2} + \frac{1}{1.2.3} + \frac{1}{1.2.3.4} + \dots \text{till } \infty \text{ terms}$$

$$\lambda = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} - \log n \right)$$

The irrationality of such numbers as $\sqrt{2}, \sqrt{3}, 2^e, \pi^e, \pi^{\sqrt{2}}$ or 'Euler's constant' γ is still unproved. Here-

$\log x=y$ means that $e^y=x$

There are many other irrational numbers. Actually they are much more than rational numbers, this is higher infinity. The rational numbers not classified by any algebraic equation are called Transcendental.

All the numbers-natural, rational and irrational-are jointly called Real numbers. Compared to them, there are imaginary numbers. Square of positive or any negative number is always positive. E.g.

$$2 \times 2 = 4, \quad (-2) \times (-2) = 4.$$

Thus there is no number whose square-root is a negative number. The number which we define as square-root of a negative number is an imaginary number. All imaginary numbers are written as square-root of (-1) called i -

$$\sqrt{-1} = i$$

Thus $\sqrt{-4} = 2i$ and so on. A combination of real and imaginary number is called a complex number.

Number of points in any line is equal to real numbers and numbers on a plane, or two perpendicular straight lines is equal to complex numbers. E.g. $(2+3i)$ is a point in a plane which is 2 units towards right from origin and 3 units up. Negative numbers are towards left or below origin. Its theory and use can be seen in any book on complex variable.

$$-\infty < \dots -3, -2, -1, 0, 1, 2, 3, \dots +\infty$$

Infinity is defined to be a quantity which is greater than any number we can choose. The last limit in positive direction is positive infinity and in negative direction, it is negative infinity. Infinity is not a number, but greater than any number.

7. Grades of Infinity-Collection of objects is called a set. Set of all natural numbers is written as

$$N = \{1, 2, 3, \dots\}$$

It appears that set of all positive integers $N = \{1, 2, 3, \dots\}$ is larger than the set $\{2, 4, 6, \dots\}$ of all even integers. But it is not so. For each member of the first set, there is a separate number in the second set, so both are equal. In another way, double of infinity (members in second set) is also infinity. Matching of each member of the sets can be shown by putting the pairs one below the other-

$$1, 2, 3, \dots, n, \dots$$

$$2, 4, 6, \dots, 2n, \dots$$

In the same way we can show that N is numerically equivalent to the set of all even integers (positive and negative)-

$$1, 2, 3, 4, 5, 6, 7, \dots$$

$$0, 2, -2, 4, -4, 6, -6, \dots$$

Similarly, N is numerically equivalent to set of all integers-

$$1, 2, 3, 4, 5, 6, 7, \dots$$

$$0, 1, -1, 2, -2, 3, -3, \dots$$

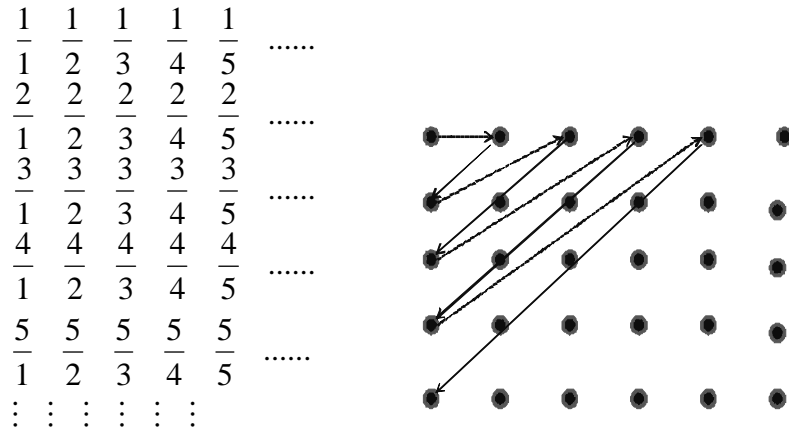
In early 17th century, Galileo had shown that there are as many perfect squares $(1, 4, 9, 16, 25, \text{etc.})$ as there are positive integers, which appeared strange to him-

$$1, 2, 3, 4, 5, 6,$$

$$1^2, 2^2, 3^2, 4^2, 5^2, \dots$$

In 19th century, Georg Cantor (1845-1918 AD) of Jewish descent born in St. Petersburg, Russia showed that set of all positive fractions (rational numbers) is equal to set of cardinal numbers. To prove it, we have to devise a method of counting

in which no positive fraction is left out. Cantor arranged all possible fractions in two rows and counted them in diagonal manner, starting from top left hand corner. His scheme is shown below.



Thus the counting will be-

1, 1/2, 2, 1/3, 3, 1/4, 2/3, 3/2, 4, 1/5, 5,

Cantor further proved that, if this countable infinity is N_0 , then the real number set is much bigger than natural numbers. That is not countable and is equal to 2^{N_0} . Set of real numbers is

$$2^{N_0} = c$$

Consider two sets- $N = \{1, 2, 3, \dots\}$ and $I = [0, 1)$, the first with cardinal number (count) N_0 and the second with cardinal number c . If N is class of all subsets of N_0 , then N is countable by c . N is class of all sub sets of N_0 , each member can be kept or omitted, so there will be two subsets for each, and total subsets are 2^{N_0} . If A is a subset of N , then $f(A)$ is that real

number x in I whose decimal expansion $x = .d_1 d_2 d_3 \dots$ is defined by the condition that d_n is 3 or 5 according as n is or is not in A . Any other two digits can be used here, as long as neither of them is 9. If x is a real number in I and if $x = .b_1 b_2 b_3 \dots$ is its binary expansion (so that each b_n is either 0 or 1), then it is that subset of n which includes all members for which $b_n = 1$.

By similar logic set of all subsets of real numbers is equal to 2^c and we get a chain of cardinal numbers containing many grades of infinity-

$$1 < 2 < 3 < \dots < N_0 < c < 2^c < 2^{2^c} < \dots$$

8. Indian classes of Infinity-There are many names of *Viṣṇu* meaning infinity in *Viṣṇu-sahasranāma-Ananta* (659, 886), *Anantajit* (307), *Anantarūpa* (932), *Anantaśrī* (933), *Anantātmā* (518), *Aniruddha* (185, 638), *Anirdeśyavapu* (177, 656), *Anekamūrti* (721), *Apām-nidhi* (323), *Avyaya* (13, 900), *Aprameya* (46), *Aprameyātmā* (248), *Amānī* (747), *Amitavikrama* (516, 641), *Ameyātmā* (102, 179), *Ambhonidhi* (517), *Asaṅkhyeya* (247), *Asammita* (108), *nidhih avyayah* (30), *Naikah* (726), *Naikakarmakṛt* (469), *Naikajah* (890), *Naikamāyah* (302), *Naikarūpah* (271), *Naikaśṛṅgah* (763), *Naikātmā* (468), *Paramātmā* (11), *Parameṣṭhī* (419), *Parardhih* (389), *Parigraha* (420), *Paryavasthitah* (931), *Pūrṇah* (685), *Bṛhat* (836), *Brahma* (663, 664), *Brahmavivardhanah* (665), *Brahmaṇya* (669), *Brahmī* (668), *Mahat* (841), *Mahardhi* (350), *Mahākramah* (671), *Mahānidhi* (806), *Mahāmāyah* (170), *Mahārha* (522), *Viśvam* (1), *Sarvah* (25). In addition, some other words also may mean infinity-*sahasra* (1000, infinity), *Vīra* (brave, boundary, beyond boundary or *Akabar* in Arabic).

Śulba-sūtra (rule of measuring structures) also indicate words for different types of infinity-

अपरिमितं प्रमाणाद्भूयः । (कात्यायन शुल्ब सूत्र १/२३)

I.e. *aparimita* means greater than a standard or limit.

It is difficult to define the terms of *Viṣṇu-sahasranāma* in exact mathematical terms. The same word in different places may have different meanings-

Ananta has been defined differently by *Śaṅkarāchārya* at 2 places-At serial 659 he tells-

व्यापित्वान्नित्यत्वात् सर्वात्मत्वात् देशतः कालतो वस्तुतश्चापरिच्छिन्नः,
अनन्तः-सत्यं ज्ञानमनन्तं ब्रह्म (तैत्तिरीय उपनिषद् २/१) इति श्रुतेः ।

गन्धर्वाप्सरसः सिद्धाः किन्नरोरगचारणाः ।

नान्तं गुणानां गच्छन्ति तेनानन्तोऽयमव्ययः ॥ (२/५/२४)

इति विष्णु पुराण वचनाद्वा अनन्तः ।

At serial 886 he tells-

नित्यत्वात् सर्वगतत्वाद् देशकालपरिच्छेदाभावात् अनन्तः शेषरूपो वा ।

At both places, these meanings are indicated-eternal, all pervading, soul of all, not bound by time and space, *satya* (truth, sameness) of three types etc. At first place, it occurs after words-*Anirdeśya-vapu*=indeterminable body, *Viṣṇu*=enclosing all, so it means infinite in time and space. At second place, it occurs with *Hutabhuk*, *Bhoktā* =consumer etc. Here, it may mean infinite consumption or work.

Asaṅkhyeya means not measurable with cardinal numbers.

Aprameya is numbers not defined with algebraic formula.

Ambhonidhi is collection of continuous numbers like spread of water in 3 dimensional space.

Naikah means not measurable with cardinal numbers starting with 1.

Jaina texts have given many forms of infinity as per -Tao of *Jaina Sciences*-by Prof. Laxmi Chandra Jain-in appendix 2 (published by Arihanta Publications, Punjabi Bag, Delhi-

Samkhyā pramāṇa (The Number Measure)

<i>Samkhyāta</i> (Numerator)	<i>Asamkhyāta</i> (Innumerate)	<i>Ananta</i> (Infinite)
Minimal Medium Maximal		
<i>Parita</i>	<i>Yukta</i>	<i>Asamkhyāta</i>
<i>Asamkhyāta</i>	<i>Asamkhyāta</i>	<i>Asamkhyāta</i>
Minimal Medium Maximum		
Minimal Medium Maximum		
<i>Parita</i>	<i>Ananta</i>	<i>Yukta</i>
Minimal Medium Maximal	Minimal Medium Maximal	<i>Ananta</i>
	Minimal Medium Maximal	<i>Ananta</i>

Without examples, it is difficult to surmise about the definitions. The original authors must be having idea about different types of infinity.

Chapter 2

Number System

1. Numbers in decimal system-We are using 9 number signs and one 0 to write any number in decimal system. Any number from 0 to 9 can be written by these symbols. To write number bigger than 9, we put 1 at a place left to it whose value is 10 times. Thus 1 at second place means $1 \times 10 = 10$, then we increase numbers at first place from 0 to 9. When symbols are exhausted we put 2 at second place meaning $2 \times 10 = 20$ and again put 0 to 9 at first place. This place is continued till we keep 9 at second place. When there are 9 and 9 at both places, next number is written by keeping 1 at third place having value of $10 \times 10 = 100$, and keeping blank or zero at places 1 and 2. Keeping 1 at third place we again keep 01 to 99 at first two places. Thereafter, 2 is kept at 3rd place and numbers 00 to 99 are placed in two places. These 3 places can be used to write any number from 100 to 999, then we write 1000 to 9999 by using 4 places. Fourth place to left has value $10^3 = 1000$ times, 5th place number has value $10^4 = 10,000$ times and so on. Thus any number howsoever large can be written by this method. This is called decimal-system.

Example of first 100 numbers-

0, 1, 2, 3, 4, 5, 6, 7, 8, 9,
10, 11, 12, 13, 14, 15, 16, 17, 18, 19
20, 21, 22, 23, 24, 25, 26, 27, 28, 29,
30, 31, 32, 33, 34, 35, 36, 37, 38, 39,
40, 41, 42, 43, 44, 45, 46, 47, 48, 49,

50, 51, 52, 53, 54, 55, 56, 57, 58, 59,
60, 61, 62, 63, 64, 65, 66, 67, 68, 69,
70, 71, 72, 73, 74, 75, 76, 77, 78, 79,
80, 81, 82, 83, 84, 85, 86, 87, 88, 89,
90, 91, 92, 93, 94, 95, 96, 97, 98, 99.

Next numbers of 3 digits are as follows-

100, 101, 102, 103, 104, 105, 106, 107, 108, 109,

....

190, 191, 192, 193, 194, 195, 196, 197, 198, 199,

....

990, 991, 992, 993, 994, 995, 996, 997, 998, 999.

Then the numbers will start with 1000. After 9999, the 5 digits will start with 10,000 and will go upto 99,999. By this process we can form any number of any digit.

There are other systems also for writing numbers used for special purposes. These will be discussed later on.

2. Vedic origin of decimal system-Decimal system was selected for common symbols of numbers due to 2 reasons-

1. 10 dimensional space-Real world is 10 dimensional. Space has 3 dimensions, called length, width, height. On earth surface, these are latitude, longitude, and height (above mean sea level). Similarly, 3 numbers are used to indicate position of object from earth in sky-*rāśi* (longitude), *krānti* (latitude), and *karna* (distance from earth). An event in space is indicated by indicating its time from a reference point. In Special theory of relativity, Albert Einstein showed in 1905 that time and space are related and the world is a Space-Time-Continuum. In General Theory of Relativity, he showed in 1920 that mass

is an effect of curvature of this 4-dimensional world. Later on theories of 5 to 11 dimensional spaces were made. Most common are 10-dimensional string theories-there are 5 versions of that and an improvement called M-string theory. However, physical nature of these dimensions are not clear-these are only manipulations of differential equations and special geometries.

In Vedic literature, there are distinct names and concepts for these dimensions. *Brahmā* with 4 heads is collection of all matter in space, being the creator. Thus, 4th dimension is matter. *Rudra* of 5 heads is called *Mahākāla*, so time (*kāla*) is the 5th dimension. For mechanical world, this is sufficient description. Its implication is 5 basic units of measurements in physics, indicated by 5 *Mā* (verb *mā*=to measure) *chhandas* in vedas. This is described by *Sāṅkhya* philosophy with 5x5 elements and a corresponding script (Roman) for that. Sixth element is *Chetanā* (consciousness) which does *chayana* (arrangement of objects at their places). Philosophy of this is *Śaiva* with 6x6 elements. There are $6^2 - 5^2 = 11$ extra elements equal to 11 forms of *Rudra*. Latin, Russian, Arabic and *Gurumukhi* scripts are parallel to this. Seventh element can be called *Ṛṣi* (*rassi* in Hindi=string). This is link between separate objects. The man linking the knowledge of space and passing to common persons has been called *ṛṣi* as he is 2-way link. For this space there are 7x7 *maruts* (levels of wind) and *Deva-nāgarī* script by *Indra* prepared with help of *Vāyu* (air). Eighth dimension is *Nāga* (indicating number 8) or *Vṛtra*, which creates curling effect due to which all objects have local field only. The script of this is *Brāhmī* with 8x8 elements for same number of *kalā*.

Ninth dimension is *Randhra* (hole), or *Nanda* (both indicating number 9) i.e. deficiency. Due to that creation occurs-
नवो नवो भवति जायमानः (ऋक् १०/८५/१९)

= From the ninth, creation occurs

Tenth dimension is *Ānanda* which is *Rasa* or the uniform material spread in root form of world. Script of 9th dimension is *vijñāna-vāk* of *vedas* with $(8+9)^2 = 289$ letters/symbols-
36x3=108 vowels, 36x5=180 consonants and 1 un-attached *Om*. Script for 10 dimensions is Chinese with thousand letters-
सहस्राक्षरा परमे व्योमन् । (ऋक् १/१६४/२५)

= *Vāk* (word) is of thousand letters in *parama* (beyond) *vyoma* (sky). *Vyoma* on earth is *Trivṛtapa* (Tibet)=heaven, beyond that in China, script has 1000 letters.

2. Easy calculation-Man has 10 fingers which help in 10 digit counting. Actually, digit means both-a number, a finger. In fractions with factor 2 or 5 in denominator, decimal will close (non-recurring). In system of 7, all decimals except division by 7 will be recurring. In 8-system, only divisors of factor 2 ($8=2 \times 2 \times 2$) will give non-recurring decimal. Similarly, 9-system will be divisible only by factors of 3. Some mathematicians preferred 12-system which will be divisible by 2, 3. But this will need 12 symbols 1 to 11 and a zero- this is more than number of fingers in hands. Base of 10 is equally useful.

3.Vedic references-

स्थानात्स्थानं दशगुणमेकस्माद्गण्यते द्विज ।

ततोऽष्टादशमे भागे परार्द्धमभिधीयते । (विष्णु पुराण ६/३/४)

O *brāhmaṇa*! from one place to the next one, the places are

multiples of ten. The 18th number is called *parārdhha*.

एकं दश शतं चैव सहस्रायुतलक्षकम् ।।

प्रयुतं कोटिसंज्ञां चार्बुदमब्जं च खर्वकम् ।

निखर्वं च महापद्मं शङ्कुर्जलधिरेव च ।।

अन्त्यं मध्यं परार्द्धं च संज्ञा दशगुणोत्तराः ।

क्रमादुत्क्रमतो वापि योगः कार्योऽन्तरं तथा ।।

(बृहन्नारदीय पुराण २/५४/१२-१४)

The numbers successively bigger ten times are-*eka* (1), *daśa* (10), *śata* (100), *sahasra* (1000), *ayuta* (10,000), *lakśa* (10⁵), *prayuta* (10⁶), *koṭi* (10⁷), *arbuda* (10⁸), *abja* (10⁹), *kharva* (10¹⁰), *nikharva* (10¹¹), *mahāpadma* (10¹²), *śaṅku* (10¹³), *jaladhi* (10¹⁴), *antya* (10¹⁵), *madhya* (10¹⁶), *parārdhha* (10¹⁷).

एकदशशतसहस्रायुतलक्षप्रयुतकोटयः क्रमशः ।

अर्बुदमब्जं खर्वनिखर्वमहापद्मशङ्कुवस्तस्मात् ।।

जलधिश्चान्त्यं मध्यं परार्द्धमिति दशगुणोत्तराः संज्ञाः ।

संख्यायाः स्थानानां व्यवहारार्थं कृताः पूर्वेः ।।

(लीलावती, परिभाषा १०-१२)

अयुतं प्रयुतं चैव शङ्कुं पद्मं तथार्बुदम् ।

खर्वं शङ्कुं निखर्वं च महापद्मं च कोटयः ।।

मध्यं चैव परार्द्धं च सपरं चात्र पण्यताम् ।

(महाभारत, सभापर्व ६५/३-४)

All these give the same sequence of numbers. *Mahābhārata* adds *para* (10¹⁸) after *parārdhha*.

यदर्धमायुषस्तस्य परार्द्धमभिधीयते ।

(श्रीमद् भागवत पुराण ३/११/३३)

Half-life of *Brahmā* is *parārdhha*.

परार्द्धद्विगुणं यत्तु प्राकृतस्स लयो द्विज ।

(विष्णु पुराण ६/३/४)

Prakṛti merges into it (*Brahmā*) after 2 *parārdhha*.

लौकिके वैदिके वापि तथा सामयिकेऽपि यः ।

व्यापारस्तत्र सर्वत्र संख्यानमुपयुज्यते ।।९ ।।

बहुभिर्विप्रलापैः किं त्रैलोक्ये सचराचरे ।

यत्किञ्चिद् वस्तु तत्सर्वं गणितेन विना नहि ।।१६ ।।

(महावीराचार्य, गणितसारसंग्रह १/९,१६)

In worldly life or Vedic matters, or even in religious practices, whatever be the dealings, everywhere enumeration is essential. Why say much ? In the three worlds, living or non-living, whatever is to be transacted, that cannot be done without calculation.

एकं दश शतं च सहस्रं त्वयुतनियुते तथा प्रयुतम् ।

कोट्यर्बुदं च वृन्दं स्थानात् स्थानं दशगुणं स्यात् ।

(आर्यभट-१, आर्यभटीय २/२)

एकं दश च शतं चाथ सहस्रमयुतं क्रमात् ।

नियुतं प्रयुतं कोटिरर्बुदं वृन्दमप्यथ ।।५ ।।

खर्वो निखर्वश्च महापद्मः शङ्कुश्च वारिधिः ।

अन्त्यं मध्यं परार्द्धं च संख्या दशगुणोत्तराः ।।६ ।।

(शङ्करवर्मन्, सद्गुणमाला, १/५-६)

Āryabhaṭa-1 and *Śaṅkaravarman* have listed the same sequence upto *parārdhha*, however, 10⁵ has been called *niyuta*, and 10⁹ called *Vṛnda*.

इमा म अग्न इष्टका धेनवः सन्तु-एका च दश च, दश च शतं च, शतं च सहस्रं च, सहस्रं चायुतं चायुतं च नियुतं च, नियुतं च प्रयुतं च, अर्बुदं च न्यर्बुदं च, समुद्रश्च मध्यं चान्तश्च परार्धश्चैता मे अग्न इष्टका धेनवः सन्त्वमुत्रा मुष्मिल्लोके । (वाजसनेयी यजुर्वेद १७/२)

O *Agni*, may these bricks be mine cows (givers of desired objects)-1 and 10, 10 and 100, 100 and 1000, 1000 and 10^4 , 10^4 and 10^5 , 10^5 and 10^6 , 10^7 , 10^8 , 10^9 , 10^{10} , 10^{11} , 10^{12} , 10^{13} , 10^{14} , 10^{15} , 10^{16} and 10^{17} . May these be my cows in this and next world.

अथार्जुनो गणको महामात्रो बोधिसत्त्वमेवमाह-जानीषे त्वं कुमार कोटि शतोत्तरां नाम गणनाविधिम्?

आह-जानाम्यहम् ।

आह-कथं पुनः कोटिशतोत्तरा गणना गतिरनुप्रवेष्टव्या?

बोधिसत्त्व आह-शतमयुतानां नियुतं नामोच्यते । शतं नियुतानां कङ्कारं नामोच्यते । शतं कङ्काराणां विवरं नामोच्यते । शतं विवराणां अक्षोभ्यं नामोच्यते । शतमक्षोभ्याणां विवाहं नामोच्यते । शतं विवाहानां उत्सङ्गं नामोच्यते । शतमुत्सङ्गानां बहुलं नामोच्यते । शतं बहुलानां नागबलं नामोच्यते । शतं नागबलानां तिटिलम्भं नामोच्यते । शतं तिटिलम्भानां व्यवस्थानप्रज्ञप्तिर्नामोच्यते । शतं व्यवस्थानप्रज्ञप्तीनां हेतुहिलं नामोच्यते । शतं हेतुहिलानां करहूनामोच्यते । शतं करहूणां हेत्विन्द्रियं नामोच्यते । शतं हेत्विन्द्रियाणां समाप्तलम्भं नामोच्यते । शतं समाप्तलम्भानां गणनागतिर्नामोच्यते । शतं गणनागतीनां निरवद्यं नामोच्यते । शतं निरवद्यानां मुद्राबलं नामोच्यते । शतं मुद्राबलानां सर्वबलं नामोच्यते । शतं सर्वबलानां विसंज्ञागतिर्नामोच्यते । शतं विसंज्ञागतीनां सर्वसंज्ञा नामोच्यते । शतं सर्वसंज्ञानां विभूतङ्गमा नामोच्यते । शतं विभूतङ्गमानां तल्लक्षणं नामोच्यते ।

(ललितविस्तर १६८-६९)

The mathematician, minister *Arjuna* asked *Bodhisattva*-O prince, do you know the counting which goes beyond hundred *koṭi* (in the centesimal scale)?

Bodhisattva-I know.

Arjuna-How does the counting proceed beyond hundred *koṭi* (in the centesimal scale)?

Bodhisattva-Hundred *koṭis* make one *ayuta*, hundred *ayutas* make one *niyuta*, hundred *niyutas* make one *kaṅkāra*, hundred *kaṅkāras* make one *vivara*, hundred *vivaras* make one *akśobhya*, hundred *akśobhyas* make one *vivāha*, hundred *vivāhas* make one *utsaṅga*, hundred *utsaṅgas* make one *bahula*, hundred *bahulas* make one *nāgabala*, hundred *nāgabalas* make one *tiṭilambha*, hundred *tiṭilambhas* make one *vyavasthānaprajñāpti*, hundred *vyavasthānaprajñāptis* make one *hetuhila*, hundred *hetuhilas* make one *karahū*, hundred *karahūs* make one *hetvīndriya*, hundred *hetvīndriyas* make one *samāptalambha*, hundred *samāptalambhas* make one *gaṇanāgati*, hundred *gaṇanāgatis* make one *niravadya*, hundred *niravadyas* make one *mudrābala*, hundred *mudrābalas* make one *sarvabala*, hundred *sarvabalas* make one *visaṇjñāgati*, hundred *visaṇjñāgatis* make one *sarvasaṁjñā*, hundred *sarvasaṁjñās* make one *vibhūtaṅgamā*, hundred *vibhūtaṅgamās* make one *tallakśaṇa* (i.e. 10^{53}).

4. Limit of *parārdha*-The example of *Lalita-vistara* given above shows that at least numbers upto 10^{53} were named in India. *Jaina* mathematics uses much bigger numbers. E.g. *Jaghanya parita asankhya* is expressed as

$\log[(kk)N]/(\log 8)$, where the integer next to N is given by 7.9656×10^{135} (nearly), and k is greater than

Details may be seen in Tao of Jain Sciences by Prof L.C. Jain, page xv or *Viśva-prahelikā* by Muni Mahendra kumar-II. Many other examples can be shown.

However, the practical number system was upto *parārdha* (10^{17}) only due to two reasons-

(1) Size of galaxy is $10^7 \times 10^7 = 10^{14}$ times that of earth. *Viṣṇupurāṇa* (2/7/4) tells that whatever is the size of earth starting from man, the same is the size of its sky compared to earth-

यावत्प्रमाणा पृथिवी विस्तार परिमण्डलात् ।

नभस्तावत्प्रमाणं वै व्यासमण्डलतो द्विज ॥४॥

The previous verse defines the 3 earths mentioned in *vedas*-3 earths and their skies are 3 mothers and 3 fathers (e.g. *Rk* 1/164/10, 2/27/8, 10/81/5 etc.). Each earth is the zone lighted by sun and moon. The planet earth is lighted by sun-moon both. The zone lighted by sun is the second earth called *Maitreya-maṇḍala*, its 3 zones of heat, brightness (*loka* part), light are 3 steps of *Viṣṇu*, Last limit of sun rays is galaxy-at that distance sun is seen just like a point star. This is extreme limit or *Parama-pada* of *Viṣṇu*-

रवि चन्द्रमसोर्याविन्मयूखैरवभास्यते ।

स समुद्र सरिच्छैला तावती पृथिवी स्मृता ॥३॥

The earth itself as a thousand petalled lotus is of 1000 *yojanas* in diameter. Thus galaxy is of 10^{17} *yojanas* and has been called *parama-guha* (largest cave) of *parārdha* size.

ऋतं पिबन्तौ सुकृतस्य लोके गुहां प्रविष्टौ परमे परार्द्धे ।

(कठोपनिषद् १/३/१)

पञ्चपादं पितरं द्वादशकृतिं दिव आहुः परे अर्धे पुरी षिणम् । (ऋक् १/१६४/१२)
Thus, numbers upto 10^{17} are sufficient to measure galaxy. Hence *Āryabhaṭa-I* and Mexican astronomy-both have used only 18 digit numbers.

(2) Life of *Brahmā* is 2 *parārdha*. His day is called *Kalpa* of 1000 *yugas*-

सहस्र युग पर्यन्तमहर्षद् ब्रह्मणो विदुः ।

रात्रिं युगसहस्रान्तां तेऽहोरात्रविदो जनाः ।१७।

अव्यक्ताद् व्यक्तयः सर्वाः प्रभवन्त्यहरागमे ।

रात्र्यागमे प्रलीयन्ते तत्रैवाव्यक्त संज्ञके ।१८। (गीता, अध्याय ८)

Day of *Brahmā* is of 1000 *yuga*. Night also is of 1000 *yuga* stated by knowers of day-night. When day comes, all creation manifests from formless. During night, it merges in the same formless. With 1000 *yuga* (each of 12000 *divya* years of 360 solar years) day and similar night, year of *Brahmā* is of 12×30 days and his life is of 50 years. Thus his half- life completed so far in his 50 years is $= 50 \times 12 \times 30 \times 2 \times 1000 \times 12000 \times 360 = 1.5 \times 10^{17}$ years. Thus *parārdha* is sufficient in this way also.

3. Other Systems-There are references or usage of other systems also for special purposes.

(1) **Binary system**-In modern times, this is being used for mathematical logic-described by Boolean Algebra in 1840. This is the number system used in modern computers also. These work on valves- which either stop or allow the electric current. Flow is 1 and stopping is 0. This system uses only these two

$10^{10^{10^{45}}}$

digits and place value of number increases 2 times for each place to the left. For example, 10 of decimal system is 1010 in binary= $1 \times 2^3 + 0 \times 2^2 + 1 \times 2 + 0 = 8 + 0 + 2 + 0$.

For cosmic measurement in vedas, exponential system with base 2 has been used. Its reasons may be following-

(1) *Manotās* (creative causes) for *Svayambhū maṇḍala* are- *veda* (knowledge), *sūtra* (=thread, link between different points), and *niyati* (destiny, future, mental attitude). Due to two types of *sankalpa*-there are two chains of creation and dissolution-called *sañchara* and *prati-sañchara* in *sāṅkhya* philosophy; or *Sambhūti* (creation) and *Asambhūti* or *vināśa* (destruction) in *Īśāvāsyopaniṣd* (Chap 40 of *Yajurveda*). Thus, structures of space should be measured in scale of two.

(2) Our perception of sound or light energy is logarithmic as per Helmholtz principle in modern physics. Thus measurement of sound energy is in logarithmic scale Bel (tenth part is decibel). Similarly, earthquake intensity is measured in logarithmic scale. Vice versa, real energy of earthquake, light, or sound is in exponential scale of our perception. In modern mathematics, we take base of e which is equal to

$$e = 1 + \frac{1}{1} + \frac{1}{1 \times 2} + \frac{1}{1 \times 2 \times 3} + \dots \dots \text{to } \infty$$

However, it appears that real structure of universe corresponds to measurement with base 2.

Measures in this scale are called-*ahar*, and their collection will be *ahargaṇa*. Division by 2 is called *horā* which is short form of *aho-rātra* (day-night). Its reverse *ahar* should thus mean double. Measure of space has been called *ahar* at many places-

त्रिंशद्द्वाम वि राजति वाक् पतङ्गाय धीयते । प्रति वस्तोरहद्युभिः ।
(ऋक् १०/१८९/३)

I.e. We measure (understand) influence of sun ruling up to 30 *dhāma*, each zone (*vastu*=object, zone in sky) in sky (*dyu*) is measured by *ahar* (-*gaṇa*=count)

अहर्वै वियच्छन्दः (यजु.१५/५)-शतपथ ब्राह्मण (८/५/२/५)

Ahar (referred in *Yajurveda* 15/5) is *chhanda* (measure) of *Viyat* (vacant, space)

अहर्वै विष्णुक्रमाः । शतपथ ब्राह्मण (६/७/४/१२)

Ahar are measures of steps of *Viṣṇu*.

See also *Rk* (1/132/2,3), (1/32/2), (2/11/5), (1/103/2), (5/29/3), (3/32/11), (4/19/2), (6/30/4), (4/28/3) etc. and other *vedas*.

Each *ahar* is double of previous *ahar*-

द्वात्रिंशतं वै देवराथाह्यान्ययं लोकः । तं समन्तं पृथिवी द्विस्तावत्पर्येति । तां समन्तं पृथिवीं द्विस्तावत्समुद्रः पर्येति । (बृहदारण्यक उपनिषद् ३/३/२, अर्थात् शतपथ ब्राह्मण १४/३/१/२)

This *loka* of *Deva-ratha* (=chariot or body of *devas* or lighted zones) is of 32 *ahar* (*ahāni*=plural of *ahar*). All around it is the earth (solar system of 33 *ahar* in this context) which is double in size. Again double sphere around this (34th *ahar*) is sea (spread of galactic matter).

Earth is the measuring rod for such measure and there are 3 zones within it-

मा छन्दः तत् पृथिवी, अग्निः देवता । (मैत्रायणी संहिता २/१३/१४, काठक संहिता ३९/३९-४०)

अस्तभ्नाद् द्यामसुरो विश्ववेदा अमिमीत वरिमाणं पृथिव्याः (ऋक् ८/४२/१, वा.यजु.४/३०, तै.सं.१/२/८/१)

The *Asura* (*Varuṇa*) who knows the world, measured it with earth as measuring rod.

यस्य भूमिः प्रमा अन्तरिक्षमुतोदरम् । (अथर्व १०/७/३२)

(For *JyeṣṭhaBrahma*-of galaxy)-whose measure is earth and *antarikṣa* is upper space.

मा छन्दः तत् पृथिवी, अग्निर्देवता (आपस्तम्ब श्रौत सूत्र १६/२८/१)

Mā is *chhanda* (measure), that is earth, its *devatā* is *Agni*.

पृथिव्यामिमे लोकाः (पृथिवी, अन्तरिक्ष द्यौ) प्रतिष्ठिताः ।

(जैमिनीय उपनिषद् ब्राह्मण १/१०/२)

Within earth, these *lokas* are located (earth, intermediate, sky).

तस्या एतत् परिमितं रूपं यदन्तर्वेदि (भूपिण्डः) अथैष भूमाऽपरिमितो यो बहिर्वेदिः (महापृथिवी) - ऐतरेय ब्रा. (८/५)

This limited form of that earth is internal *vedi* (creation place=earth), the unlimited bigger is outer *vedi* (grand earth).

Thus radius *d* of any *ahargaṇa* *n* is given by

$$d = r \times 2^{(n-3)}, \text{ where } r = \text{radius of earth.}$$

In this measure, solar system is 33 *ahargaṇa*-3 zones within earth and 30 beyond it. Thus size of solar system is 2^{30} of earth size, each zone successively double is a *dhāma*. *Prāṇa* or energy of each zone is a *devatā*. For 33 zones, there are 33 *devatā*. *Ānanda* (*rasa* or the original source material of universe) increases at each higher or outer zone (*Taittirīya upaniṣad*, *Bhṛgu vallī*) called *lokas*.

सैषाऽऽनन्दस्य मीमांसा भवति । युवा स्यात्साधुयुवाध्यापक आशिष्ठो द्रधिष्ठो बलिष्ठस्तस्येयं पृथिवी सर्वा वित्तस्य पूर्णा स्यात् । स एको मनुष्य आनन्दः । ते ये शतं मानुषा आनन्दाः । स एको मनुष्यगन्धर्वाणामानन्दः ।.... ते ये शतं

मनुष्यगन्धर्वाणामानन्दाः । स एको देवगन्धर्वाणामानन्दः ।... ते ये शतं देवगन्धर्वाणामानन्दाः । स एकः पितृणां चिरलोकलोकानामानन्दः ।.. ते ये शतं पितृणां चिरलोकलोकानामानन्दाः । स एक आजानजानां देवानामानन्दः ।..ते ये शतमाजानजानां देवानामानन्दाः । स एकः कर्मदेवानामानन्दः ।... ते ये शतं कर्मदेवानां देवानामानन्दाः । स एको देवानामानन्दः ।.. ते ये शतं देवानामानन्दाः । स एक इन्द्रस्यानन्दः । ते ये शतमिन्द्रस्यानन्दाः । स एको बृहस्पतेरानन्दः ।..ते ये शतं बृहस्पतेरानन्दाः । स एकः प्रजापतेरानन्दः ।..ते ये शतं प्रजापतेरानन्दः । स एको ब्रह्माण आनन्दः । (२/८)

This is estimate of *ānanda*-it increases 100 times each time for the *loka* (zones) of *manuṣya* (man), *manuṣya-gandharva*, *deva-gandharva*, *pitar-loka*, *Ājānaja-deva*, *karma-deva*, *Deva*, *Indra*, *Bṛhaspati*, *Prajāpati*, *Brahmā*.

(2) **Seven system**-These are formed due to 7 tongues of *agni* (matter or energy in a closed boundary)-

काली कराली च मनोजवा च सुलोहिता या च सुधूम्रवर्णा ।

स्फुलिङ्गिनी विश्वरुची च देवी, लेलायमाना इति सप्तजिह्वाः (१/२/४)

Agni has 7 tongues for intake-*kālī*, *karālī*, *manojavā*, *sulohitā*, *sudhūmravarṇā*, *sphulinginī*, *viśvaruchī*. (*Muṇḍaka upa.*)

सप्तप्राणाः प्रभवन्ति तस्मात् सप्तार्चिषः समिधः सप्त होमाः ।

सप्त इमे लोका येषु चरन्ति प्राणा गुहाशया निहिताः सप्त सप्त (२/१/८) ।

From these 7 tongues are created-7 *prāṇa*, 7 *archi* (flame), 7 *samidhā* (firewood, implement), 7 *homa* (consumption), 7 *loka*, 7-7 cave spaces.

सप्त युज्जन्ति रथमेकचक्रमेको अश्वो वहति सप्तनामा ।

त्रिणाभिचक्रमजरमनर्व यत्रेमा विश्वा भुवनानि तस्थुः । (ऋक् १/१६४/२)

7 join (*yuj*) the single *ratha* (vehicle=body of man or of cos-

mic structures), 1 *aśva* (horse, *śva* =next day, present state will not remain tomorrow , so time is called *aśva*) flow with 7 names. Eternal cycle of motion has 3 navels (center or focus) in which worlds and cosmic structures are located. Here *yuj* of 7 types forms *yuga* or *yojana*-so both are of 7 types.

इमं रथमधि ये सप्त तस्थुः सप्तचक्रं सप्त वहन्त्यश्वाः ।

सप्तस्वसारो अभि सं नवन्ते यत्र गवां निहिता सप्त सप्त । ऋक् १ / १६४ / ३

These 7-fold divisions have led to 7 divisions of length unit *āṅgula* in *Lalita-vistara*, a *Bauddha* text-

7 <i>paramāṇu-rajās</i>	=1 <i>reṇu</i>
7 <i>reṇus</i>	=1 <i>truṭi</i>
7 <i>truṭis</i>	=1 <i>vātāyana-rajā</i>
7 <i>vātāyana-rajās</i>	=1 <i>śaśa-rajā</i>
7 <i>śaśa-rajās</i>	=1 <i>aidaka-rajā</i>
7 <i>aidakas-rajā</i>	=1 <i>go-rajā</i>
7 <i>go-rajās</i>	=1 <i>likśā-rajā</i>
7 <i>likśā-rajās</i>	=1 <i>sarṣapa</i>
7 <i>sarṣapas</i>	=1 <i>yava</i> (barley corn)
7 <i>yavas</i>	=1 <i>āṅgulī-parva</i>
12 <i>āṅgulī-parvas</i>	=1 <i>vitastī</i> (span)
2 <i>vitastis</i>	=1 <i>hasta</i> (fore arm or cubit)

1 cubit may be taken as 18 inches or 45 cms. Then-

1 *āṅgula* =45/12 cm=3.75 cm.

1 *paramāṇu-rajās*=3.75x 7⁻¹⁰=1.3x10⁻⁸ cm.

In modern physics, size of atom is in same units of 10⁻⁸ cms, called angstrom.

(3) System of 8- This is the systems for small units in astronomy.

For example, *Śrīpati* has given this scale-

8 <i>trasareṇus</i>	=1 <i>reṇu</i>
8 <i>reṇus</i>	=1 <i>bālāgra</i>
8 <i>bālāgras</i>	=1 <i>likśā</i> (poppy seed)

8 *likśās* =1 *yūka* (louse)

8 *yūkas* =1 *yava* (barley corn)

8 *yavas* =1 *āṅgula*, same as above.

Thus, 1 *trasareṇu* =1 *āṅgula* x 8⁻⁶ = 1.43x10⁻⁵ cm.

Tiilo-pannati (*Trailokya Prajñapti*)-a *Jaina* text gives these measures-

Infinitely many <i>paramāṇus</i>	=1 <i>avasannāsanna skandha</i>
8 <i>avasannāsanna skandhas</i>	=1 <i>sannāsanna skandha</i>
8 <i>sannāsanna skandhas</i>	=1 <i>truṭreṇu</i>
8 <i>truṭreṇus</i>	=1 <i>trasareṇu</i>
8 <i>trasareṇus</i>	=1 <i>rathareṇu</i>
8 <i>rathareṇus</i>	=1 <i>uttama bhogabhūmi bālāgra</i>
8 <i>ut. bhogabhūmi bālāgra</i>	=1 <i>madhyama bhogabhūmi bālāgra</i>
8 <i>ma.bho. bālāgra</i>	=1 <i>jaghanya bhogabhūmi bālāgra</i>
8 <i>ja. bhogabhūmi bālāgra</i>	=1 <i>karma bhūmi bālāgra</i>
8 <i>karma bhūmi bālāgra</i>	=1 <i>likśā</i> (poppy seed)
8 <i>likśās</i>	=1 <i>yūka</i> (louse)
8 <i>yūkas</i>	=1 <i>yava</i> (barley corn)
8 <i>yavas</i>	=1 <i>āṅgula</i> (finger width)
6 <i>āṅgulas</i>	=1 <i>pāda</i> (length of human foot)
2 <i>pādas</i>	=1 <i>vitastī</i> (span)
2 <i>vitastis</i>	=1 <i>hasta</i> (fore arm or cubit)

Here, 1 *trasareṇu* =1 *āṅgula* x 8⁻⁹ = 2.7x10⁻⁸ cm.

1 *avasannāsanna skandha*=1 *āṅgula* x 8⁻¹² = 5.2x10⁻¹¹ cm.

(4) System of 12- It is used for selling fruits, counting of papers etc. In India this has been used for 12 *rāśis*, 12 months, 12 *guru*-years etc. Dozen (*darjan*)=12

(5) System of 15- This has been used by the scholar *Bārkalī* as described in *Śatapatha Brāhmaṇa* (12/3/2/5)

दश च वै सहस्राण्यष्टौ च शतानि सम्बत्सरस्य मुहूर्ताः । यावन्तो मुहूर्तास्तावन्ति पञ्चदशकृत्वः क्षिप्राणि । यावन्ति क्षिप्राणि तावन्ति पञ्चदशकृत्व एतर्हीणि ।

यावन्त्येतर्हीणि तावन्ति पञ्चदशकृत्व इदानीनि । यावन्तीदानीनि तावन्तः पञ्चदशकृत्वः प्राणाः । यावन्तः प्राणास्तावन्तोऽक्तनाः । यावन्तोऽक्तनास्तावन्तो निमेषाः । यावन्तो निमेषास्तावन्तो लोमगर्ताः । यावन्तो लोमगर्तास्तावन्ति स्वेदायनानि । यावन्ति स्वेदायनानि तावन्त एते स्तोका वर्षन्ति । ५ ।

There are 10,800 *muhūrta* in a year. *Muhūrta* divided by 15 gives *kṣīpra*, *etarhi*, *idānī*, *prāṇa*, *aktana* (or *ana*), *nimeṣa*, *lomagarta*, *svedāyana*. Rain drops fall equal to *svedāyana*.

एतद्ध स्म वै तद्विद्वानाह वार्कलिः । सार्वभौमं मेघं वर्षन्तवेदाहम् । अस्य वर्षस्य स्तोकमिति । ६ । (शतपथ १२/३/२)

Thus stated scholar *Vārkalī*. I know universal *megha* (cloud), its drops/parts of year.

एभ्यो लोमगर्तेभ्य ऊर्ध्वानि ज्योतीष्यान् । तद्यानि तानि ज्योतीषि, एतानि तानि नक्षत्राणि । यावन्त्येतानि नक्षत्राणि तावन्तो लोमगर्ताः । (शतपथ १०/४/४/२)

Number of *lomagarta* (in a year) is equal to number of *nakṣatra* (stars in galaxy).

1 *muhūrta* = 48 minutes, 1 *Kṣīpra* = 3.2 minutes, 1 *etarhi* = 12.8 seconds, 1 *Idānī* = 0.85 sec.

1 *Prāṇa* = 0.056 sec., 1 *Aktana* = 0.004 sec., 1 *Nimeṣa* = 0.0002 sec., 1 *Lomagarta* = 59326.2 parts of a second. 1 *Svedāyana* = 8,89,892.6 parts of second.

Lomagarta in a year = $10,800 \times 15^7 = 1.845 \times 10^{12}$ (cells in human body). *Puruṣa* is 10 times world, so there are 10^{11} stars in galaxy or 10^{11} galaxies in visible universe.

In *Svedāyana* time, light moves 3×10^8 meter / 8,89,892.6 = 337 mteters-From this distance rain drops fall (or retain their shape)

(6) System of 60-This is also called sexagesimal system, usually attributed to Sumerians. However, this is used in ancient texts of Indian astronomy for sub-divisions of angle and time, which run parallel. As there are about 360 days (round number) in a year (354 days of lunar and 365.25 days in solar year), it is divided into 12 months of 30 days each. 1 day is successively divided by 60 at each step. Parallel to that, a circle has 360° degrees and each degree is repeatedly divided by 60.

प्रतत्परा षष्टिगुणा हि तत्परा विलिप्तिका सैवमसो तथा कला ।

सैवं लवस्तत्त्रिंशदाहतिर्भविद् राशिः स मार्ताण्ड गुणो भमण्डलम् ॥

(शङ्कर वर्मन्, सद्रत्नमाला २/४)

60 *prataparās* = 1 *tatparā*

60 *tatparās* = 1 *vilīptikā* (*vilīptā* or *vikalā*)

60 *vilīptikās* = 1 *liptikā* (*liptā* or *kalā*)

60 *liptikās* = 1 *lava* (or *bhāga*)

30 *lavas* = 1 *rāśi*

12 *rāśis* = 1 celestial circle

गुर्वक्षरं विघटिका घटिका दिनं च, पूर्वाणि षष्टिगुणितानि निजोत्तराणि ।

त्रिंशद्गुणं दिवसमत्र च माससंज्ञः मासो दिवाकर गुणः खलु सावनाब्दः ॥

(शङ्कर वर्मन्, सद्रत्नमाला २/१)

60 *gurvakśaras* = 1 *vigṛṭikā*

60 *vigṛṭikās* = 1 *ghṛṭikā*

60 *ghṛṭikās* = 1 day

30 days = 1 months

12 months = 1 year

It may be noted that it is used only for these measures. For writing general numbers in this system, 60 symbols including

zero will be needed, which will be very cumbersome.

This is also linked to 60 year cycle for *guru* years in which jupiter makes 5 and saturn 2 revolutions. In *vedas* it has been called *Āṅgirā* year.

आदित्याश्च ह वा आङ्गिरसश्च स्वर्गे लोके स्पर्धन्त-वयं पूर्वो एष्यामो वयमिति । ते हाऽऽदित्याः पूर्वे स्वर्गं लोकं जग्मुः, पश्चेवाङ्गिरसः, षष्ठ्यां वा वर्षेषु (ऐतरेय ब्रा.१८/३/१७) ।

Āditya and *āṅgiras* were competing to reach *svarga* earlier-we will reach earlier. Those *ādityas* reached earlier, *āṅgiras* were later, in 60 years. (*Brhaspati* is son of *Āṅgiras*)

आदित्याश्चाङ्गिरसश्च सुवर्गे लोकेऽस्पर्धन्त ... त आदित्या एतं पञ्च होतार मपश्यन् (तैत्तिरीय ब्रा.२/२/३/५)

Āditya and *āṅgiras* were competing to reach *svarga* earlier. Those *ādityas* saw 5 *Hotā* (They covered 5 *hotā* or years each-years consumes world, so it is *hotā*). *Ādityas* are 12.

So, *Āditya* x 5 *hotā* = 12x5=60 years.

4. Origin of word numbers-Words indicating 1, 2....., 9, 10, 20,90, 100, 1000. etc. are since very old times and their derivation is based on their concepts. Origin of the sankrit words for numbers is given below as per old grammer and *Nirukta* - a limb of *vedas* indicating technical definition and other origins of words.

Eka (1)-This is from the root verb-*Ḥ* -meaning -to move, or to arrive. *A+i+k=Itā*=having gone or reached. The number 1 only forms all the following numbers by going ahead. *Itā* has become it in english. (*Nirukta* 3/10)

Dvi (2)-It is derived from *vi* (*kalpa*)=alternative. *Vi* has be-

come bi- suffix of greek, used in english etc. Its double number form (between singular and plural) is *dvau*- derived from *druta-tara*, meaning faster. This goes ahead of 1, i.e. the next number.

Tri (3)-*Dvi* is comparative, *tri* is superlative. It is *tīrṇatamā* = having crossed both 1 and 2. This *tīrṇatamā* has been shortened to *tri*. The Greek prefix is same-tri, this has become similar word three. Plural form is *trayah*.

Chatur (4)-*Chatvārah* is formed by *cha* = and, or more; *tvarā* = faster. This goes farther than 3 also. *Trayī* is normal division of knowledge, this means *vedas*. Their original unique source is 4th, so *trayī* means 4 *vedas*. *Chatur* goes farther than normal 3-fold class, so it means clever, efficient or expert. Hebrew/Greek quadri = qua+dri (*tri*); here also it is beyond three. This has become quart (1/4 part), quarter-also means house having 4 side walls.

Pañcha (5)-This is from word *Prkta* = joined, combined. 5 fingers are joined in single palm, 5 *mahābhūta* (source elements) combine to form the world. Word with similar meaning is *pañkti* -from verb *pach* = to make forms or classes. This is by intermixing, so it means to cook or to digest also. This has become prefix- penta in Greek. In contact consonants of sanskrit, 5th class of letters is of *pa*, so *pa* and 5 are indicated by same symbol in *Bangla* script. In Hindi also, they are similar, if we remove the head line of *pa*.

Ṣaṭ (6)-This is from word-*sahati* = suppresses (*Nirukta* 4/27). This also means lives together, or tolerates. Year is called *samvatsara* because 6 seasons live together in it (*samvasanti* = live together). This has become six, hexa- prefix, as *s* becomes

h in arabic/persian.

Sapta (7)-*Nirukta* (3/26) gives two origins-*Ṣap* = to join, to combine in war etc. *Sapta* =joined or combined. *Śrpi*= to move, to slide. *Sarpaṇa* = movement, sliding. Due to sliding motion, snake is called *sarpa*. In verb *sarpati* (=moves), *r* becomes silent- then *sapti* = horse. There are 7 horse like carriers or winds, or 7 rays of sun- so *sapta* means 7. This is same as the prefix sept, which has become seven.

Aṣṭa (8)-This is from verb *Aśū* = to pervade (*Nirukta* 3/10). Similar meaning is of the word *Vasu* =*vas+u* =which lives or pervades. 8 *vasus* are physical forms of *Śiva*, so he is called *Aṣṭa-mūrtti* (8-formed). As motion or wind, his forms are 11 *Rudras*. In astronomy texts, *vasu*=8. In Russian language, it still means 8. This has become oct- prefix and eight in english.

Nava (9)-Verb *ṇu* =to salute, or to praise. This has become *nūtana* = new. *Nava* also means both-9, new. After 9, the number system bows to 0 and new place is used for next number. Or, the 9th dimension *randhra* (hole, deficiency) is the cause of new creation (para 2/1). Thus *nava* = 9 and new. *Nirukta* (3/10) calls it short form of *Nā+va* (*nanīyā*) =not desirable, or *na+ava* (*ptā*) = not achieved. The ninth dimension or element *randhra* means something is deficient, and this is not desirable. This is origin of prefix nano- and word nine.

Daśa (10)-*Nirukta* (3/10) gives two derivations. Verb *Dasu* (*das*) =*upakśaya* = to decay. Number symbols ends after the 10th number, then new place is used. In persian, *dasta* in sense of decay (by out-flow) has two meanings-vomiting, or vomited material (this comes out of mouth, causing decay), and hand

(arising out of upper body, giving outcome of work). Another verb *Dṛśir* (*dṛś*) = to see, to look. World is seen in 10 ways, so *daśa* means ten. *Daśā* (state, condition) and *diśā* (directions) are also ten.

Viṃśati (20)-*Nirukta* (3/10) tells- *viṃśati* = *dvi+daśa+ktin* = by two tens. Alternate form *viṃśat* also has been used by *Bhāsa* in his drama *Abhiṣeka*, *Nārada-smṛti* (*divya-prakaraṇa*), and in *purāṇas*. In english, two tens = twenty.

Triṃśat or *triṃśati* (30)-Similarly, *triṃśat*=*tri+daśa+ktin* = by 3 tens. *Triṃśati* form has been used in *Pañchatantra* (5/41, 5/53) *Vimāna-śāstra* of *Bharadvāja* (page 74), *Varāha Grhya-sūtra* (6/29) etc. English form is thirty.

Chatvāriṃśat or *chatvāriṃśati* (40) = 4 tens. Second form has become forty in english.

Pañchāśat (50) = *pañcha+daśa* =5 tens. Five tens is shortened to fifty in english.

Ṣaṣṭi (60) = *ṣaṭ+daśa+ti* =by 6 tens. This has become sixty in english.

Saptati (70) =*sapta+(daśa)+ti* = by 7 tens. In English also, seventy = by seven tens.

Aśīti (80)-This is more clearly linked to verb *aśūn* (to pervade, to combine). This also gives meaning of verb *Aśa* or *Aśana*=to consume. *Aśīti chhanda* in *vedas* is measure of *anna* (grains). As per estimate of modern physics also, total number of particles in world are 10^{86} . *Aśīti* = *Aṣṭa* +(*daśa*)+*ti* = by 8 tens. In english, *aśīti* = eighty.

Navati (90) =*Nava+(daśa)+ti* = by 9 tens =ninty.

Śata (100)- *Nirukta* (3/10) gives it as short form of *daśa+daśa*.

The two middle syllables *śada* becomes *śata*. *Śatapatha Brāhmaṇa* (9/1/1/2-7) tells that enrgy of sun becomes *śānta* (calm) after 100 diameters of sun, so 100 is called *śata* (*śānta*). All activities of man also become still (*śānta*) after hundred years. *Śānta* = *Hand* = hunder or hundred of english.

Sahasra (1000)-This literally means *saha+sra* = to move together. About 1000 persons live together (one to ten thousands in a village), or move together in a batalion of army. Effect (*sāma* or *mahimā*) of sun is upto 1000 diameters, called *loka* (lighted zone). Man is known to about 1000 associates. In all these meanings, *sahasra* means 1000. In persian, it has become *hasra* = effect or result. This has further transformed to *hajāra* in hindi and other Indian languages. Sun is bright upto 1000 diameters (*sahasrākśa*) till saturn orbit, but to some extent, its effect is felt at very large distance, e.g. it is seen as a point till last limit of galaxy. So *sahasra* also means many or infinity. In greek also, word for thousand miriad has been used to indicate infinity. In any round estimate, the real number is among 1000 numbers around the estimate. Thus we do approximation upto 3 places of decimal. In this sense, *sahasra* means approximate. Like *Pururavā* had ruled for 56 years and some months, this period has been indicated about 60 years (*ṣaṣṭi sahasra varṣa*). In another method of counting *aṅka* or years of rulership, still used in Orissa, numbers ending with 0 and 6 are omitted, then *aṅka* for *Pururavā* period will be upto 64 years, which has also been indicated in some *purāṇas*.

Lakṣa (10⁵)- This is from root verb *lakṣa*=to see, to mark, to explain or to show. In this sense, all visible matters are called

Lakṣmī. Invisible property is *Vimalā* (without *mala*). We can see small objects upto about 10⁵ parts of our own size. This is called limit of resolution of human eye. So 10⁵ is called *lakṣa*. Starting from man, 7 smaller worlds are successively smaller by ratio of 10⁵ -

बालाग्र शतसाहस्रं तस्य भागस्य भागिनः ।

तस्य भागस्य भागार्द्धं तत्क्षये तु निरञ्जनम् । (ध्यानविन्दु उपनिषद्, ५)

I.e. divide hair-end by 100 thousands, and divide in same ratio 5 times again, still smaller is *Nirañjana*.

Thus, man is a world, its 1 lakh part is cell, which is world for biology. Again 1 *lakh* part is atom, which is world for chemistry. Further 1 *lakh* part is nuleus equal to inner-most layer of *Kuṇḍalinī*. Modern physics describes sizes only upto this, but *veda* goes still 4 levels lower- which are called *jagat-kaṇa*, *deva-asura*, *pitara*, *ṛṣi*. Final stage *ṛṣi* is 10⁻³⁵ meters-equal to Planck length in modern physics which is basis of string theories.

Koṭi (10⁷)-*Koṭi* means limit, end of bow, part of remaining angle till end of 1/4th circle, rise and 10⁷. E.g. limit of land-mass of India is alled *Dhanuṣkoṭi*. For man, limit (*koṭi*) of world is earth and its size is 10⁷ times human size. Hence, *koṭi*=10⁷. In same way limit of world for earth is solar system which is 10⁷ times larger. The solar system itself is earth whose sky is galaxy-again larger by 10⁷ times. The largest earth is galaxy whose sky is universe larger by 10⁷ times. (see para 2/4).

Arbuda (10⁸) and **abja** (10⁹) -*Nirukta* (3/10) defines it with a similar word-*arbuda* = *ambuda* (giver of water, cloud) as *araṇa* and *ambu*-both mean water. In example of *Śatapatha brāhmaṇa* in para 3/5, *svedāyana* has been called equal to cloud. Number

of *svedāyanas* in a *muhūrta* will be 2.56×10^9 which can be called 25 *arbuda* or 2.5 *abja*. The alternative name *vr̥nda* also means group of drops (*vindus*). Their number in a year will be much more- 2.77×10^{13} - this does not appear to be the logic. Now *araba* (*arbuda*) word is being used for 10^9 in hindi.

Kharva (10^{10}), **Nikharva** (10^{11})-In the same example it has been shown that number of stars in galaxy is 10^{11} . *Kharva* means to make powder in which particles are separated. Thus, it means number of particles as stars in galaxy. For universe, each galaxy is a particle and their number is also 10^{11} .

Mahāpadma (10^{12})- Earth is *padma*, largest earth (galaxy) is *mahāpadma* created in its field of 10 times size, having 10^{12} stars (or equal mass)-equal to number of *lomagarttas* in 1 year (para 3/5).

Śaṅku (10^{13})-*Śaṅku* (cone) or *śaṅkha* (conch) indicate rotation. The rotating field around galaxy has created it. In *chhanda* measure, it is 2^{53} of earth size, or of 56 *ahargana*. Thus *chhanda* of 56 (14x4) letters is called by similar name *śakvarī* (=night, as this is dark zone). As it creates, it is called *kūrma* (doing work, animal of its shape is also *kūrma* = tortoise) in *vedas* and *goloka* (zone of *go* =light-rays) in *purāṇas*.

मानेन तस्य कूर्मस्य कथयामि प्रयत्नतः (७) ।

शङ्कोः शतसहस्राणि योजनानि वपुः स्थितम् ।

(नरपतिजयचर्या, स्वरोदय में कूर्मचक्र)

Estimate of *Kūrma* is stated-it is hundred thousand *śaṅku* (13 zeros on 1).

शङ्कु भवत्यहो धृत्यै यद्वा अधृतं शङ्कुना तद्वाधार (११)

तद् (शङ्कु साम) उसीदन्ति इयमित्याहुः (१२)-ताण्ड्य (११/१०)

Śaṅku hold the *ahna* (collection of suns, or measured in scale

of *ahar*), as it is held by *śaṅku -sāma*, it is called *śaṅku*.

यदिमान् लोकान् प्रजापतिः सृष्ट वेदं सर्वमशक्नोद् तद् शक्नोतीति शक्नोतीति ।
(ऐतरेय ब्रा.५/७)

Śaṅku-sāma has capability (*śakva*) to create the world, so it is called *Śakvarī* also.

स यत्कूर्मो नामा एतद्वै रूपं कृत्वा प्रजापतिः प्रजा असृजत, यदसृजत अकरोत्-
तद् यद् अकरोत् तस्मात् कूर्मः (शत. ७/५/१/५)

It took the form of *Kūrma* (tortoise), and it did the work (*kurma* =we do), so it is called *kūrma*.

Jaladhi (10^{14}) - Field of galaxy or the spread of matter in it has been called *vāh* or *vāri* -meaning water as it contains all (*avāpnoti* =contains =*Ap* or *Āp* in short). As it is zone of *vāri*, its lord is *Varuṇa*.

यदवृणोत्तस्माद्वाः (जलम्)- शतपथ ब्राह्मण (६/१/१/९)

आभिर्वा अहमिदं सर्वमाप्स्यामि यदिदं किञ्चेति तस्मादापोऽभवन्, तदपामप्त्वम्
आप्नोति वै स सर्वान् कामान् । (गोपथ ब्राह्मण पूर्व १/२)

यच्च वृत्वा ऽतिष्ठन् तद् वरणो अभवत् तं वा एतं वरणं सन्तं वरुण इति आचक्षते
परोक्षेण । (गोपथ ब्राह्मण पूर्व १/७)

Size of this ocean of primordial water is 10^{14} times that of earth, which is measuring rod for universe (para 2/4). So 10^{14} has been called *Jaladhi* (spread of *jala* =water). This is square of *koṭi* (107), so it has been called *koḍākoḍī* (*koṭi* x *koṭi*) or *sāgaropama* (like ocean) in Jain astronomy.

Antya (10^{15}), **madhya** (10^{16}), **parārdha** (10^{17})-*Antya* (ending) is limit beyond *jaladhi*. *Madhya* (middle) comes between it and *parārdha*.

5. Compound words-Main words are for numbers from 1 to 10 and then in multiples of ten as shown above. All other numbers are their compounds. For example, we write 14 as 4 and ten. This is used for all numbers up to 99. The numbers ending with 9 at unit place, such as 19 are not written as nine and ten and so on. They are written as one less than twenty. *Ūna* word of sanskrit means one less, this is origin of Latin uno and english one. This is used for Roman numerals also. The chart below shows numbers in sanskrit, hindi (or similar in other Indian languages), and english forms-

Sanskrit word	Hindi form	English word
<i>ekādaśa</i>	<i>egāraha (gyāraha)</i>	eleven (elevated from ten)
<i>dvādaśa</i>	<i>bāraha</i>	twelve (two elevations)
<i>trayodaśa</i>	<i>teraha</i>	thirteen (three and ten)
<i>chaturdaśa</i>	<i>chaudaha</i>	fourteen (four and ten)
<i>pañchadaśa</i>	<i>pandraha</i>	fifteen (five and ten)
<i>ṣoḍaśa</i>	<i>solaha</i>	sixteen (six and ten)
<i>saptadaśa</i>	<i>satraha</i>	seventeen (seven and ten)
<i>aṣṭādaśa</i>	<i>athāraha</i>	eighteen (eight and ten)
<i>ūnaviṃśa</i>	<i>unnīsa</i>	nineteen (nine and ten)
<i>viṃśati</i>	<i>bīsa</i>	twenty (two tens)
<i>ekaviṃśati</i>	<i>ekkīsa</i>	twenty one
<i>dvāviṃśati</i>	<i>baīsa</i>	twenty two
<i>trayaviṃśati</i>	<i>teīsa</i>	twenty three
<i>chaturviṃśati</i>	<i>chaubīsa</i>	twenty four
<i>pañchaviṃśati</i>	<i>pacchīsa</i>	twenty five
<i>ṣaḍviṃśati</i>	<i>chhabbīsa</i>	twenty six
<i>saptaviṃśati</i>	<i>sattāisa</i>	twenty seven

<i>aṣṭāviṃśati</i>	<i>aṭṭhāīsa</i>	twenty eight
<i>ūnatrīṃśati</i>	<i>untīsa</i>	twenty nine
<i>trīṃśati (trīṃśat)</i>	<i>tīsa</i>	thirty
<i>ekatrīṃśati</i>	<i>ekatīsa</i>	thirty one
<i>dvātrīṃśati</i>	<i>battīsa</i>	thirty two
<i>trayastrīṃśat</i>	<i>taintīsa</i>	thirty three
<i>chatustrīṃśat</i>	<i>chauntīsa</i>	thirty four
<i>pañchatrīṃśat</i>	<i>paintīsa</i>	thirty five
<i>ṣaṭtrīṃśat</i>	<i>chhattīsa</i>	thirty six
<i>saptatrīṃśat</i>	<i>saintīsa</i>	thirty seven
<i>aṣṭatrīṃśat</i>	<i>aḍatīsa</i>	thirty eight
<i>navatrīṃśat</i>	<i>unchālīsa</i>	thirty nine
<i>(ūnachatvārīṃśat, ekona-chatvārīṃśat)</i>		
<i>chatvārīṃśat</i>	<i>chālīsa</i>	forty
<i>ekachatvārīṃśat</i>	<i>ikatālīsa</i>	forty one
<i>dvi (dvā)chatvārīṃśat</i>	<i>bayālīsa</i>	forty two
<i>tri (trayaś)chatvārīṃśat</i>	<i>taintālīsa</i>	forty three
<i>chatuśchatvārīṃśat</i>	<i>chauālīsa</i>	forty four
<i>pañchachatvārīṃśat</i>	<i>paintālīsa</i>	forty five
<i>ṣaṭchatvārīṃśat</i>	<i>chhiyālīsa</i>	forty six
<i>saptachatvārīṃśat</i>	<i>saintālīsa</i>	forty seven
<i>aṣṭa (aṣṭā)chatvārīṃśat</i>	<i>aḍatālīsa</i>	forty eight
<i>navachatvārīṃśat</i>	<i>unachāsa</i>	forty nine
<i>(ūnapañchāśat, ekonapañchāśat)</i>		
<i>pañchāśat</i>	<i>pachāsa</i>	fifty
<i>ekapañchāśat</i>	<i>ekāvana</i>	fifty one
<i>dvā (dvi)pañchāśat</i>	<i>bāvana</i>	fifty two
<i>trayah (tri)pañchāśat</i>	<i>tirapana</i>	fifty three

<i>chatuhpañchāśat</i>	<i>chauvana</i>	fifty four
<i>pañchapañchāśat</i>	<i>pachapana</i>	fifty five
<i>ṣaṭpañchāśat</i>	<i>chhappana</i>	fifty six
<i>saptapañchāśat</i>	<i>sattāvana</i>	fifty seven
<i>aṣṭā (aṣṭa)pañchāśat</i>	<i>aṭṭhāvana</i>	fifty eight
<i>navapañchāśat</i>	<i>unasaṭha</i>	fifty nine
<i>(ekonasaṣṭih, ūnaṣaṣṭih)</i>		
<i>ṣaṣṭih</i>	<i>sāṭha</i>	sixty
<i>ekaṣaṣṭih</i>	<i>ekasaṭha</i>	sixty one
<i>dvā (dvi) ṣaṣṭih</i>	<i>bāsaṭha</i>	sixty two
<i>trayah (tri) ṣaṣṭih</i>	<i>tirasaṭha</i>	sixty three
<i>chatuṣṣaṣṭih</i>	<i>chaunsaṭha</i>	sixty four
<i>pañchaṣaṣṭih</i>	<i>painsaṭha</i>	sixty five
<i>ṣaṭṣaṣṭih</i>	<i>chhāchhaṭha</i>	sixty six
<i>saptaṣaṣṭih</i>	<i>saḍasaṭha</i>	sixty seven
<i>aṣṭa (aṣṭā) ṣaṣṭih</i>	<i>arasaṭha</i>	sixty eight
<i>navaṣaṣṭih</i>	<i>unahattara</i>	sixty nine
<i>(ekonasaptati, ūnasaptati)</i>		
<i>saptati</i>	<i>sattara</i>	seventy
<i>ekasaptati</i>	<i>ikahattara</i>	seventy one
<i>dvā (dvi) saptati</i>	<i>bahattara</i>	seventy two
<i>trayah (tri) saptati</i>	<i>tihattara</i>	seventy three
<i>chatuhsaptati</i>	<i>chauhattara</i>	seventy four
<i>pañchasaptati</i>	<i>pachahattara</i>	seventy five
<i>ṣaṭsaptati</i>	<i>chhihattara</i>	seventy six
<i>saptasaptati</i>	<i>satahattara</i>	seventy seven
<i>aṣṭa (aṣṭā) saptati</i>	<i>aṭṭhattara</i>	seventy eight
<i>navasaptati</i>	<i>unahattara</i>	seventy nine

<i>(ekonāśīti, ūnāśīti)</i>		
<i>aśīti</i>	<i>assī</i>	eighty
<i>ekāśīti</i>	<i>ekāsī</i>	eighty one
<i>dvyāśīti</i>	<i>bayāsī</i>	eighty two
<i>tryaśīti</i>	<i>tirāsī</i>	eighty three
<i>chatuaraśīti</i>	<i>chaurāsī</i>	eighty four
<i>pañchāśīti</i>	<i>pachāsī</i>	eighty five
<i>ṣaḍaśīti</i>	<i>chhiyāsī</i>	eighty six
<i>saptāśīti</i>	<i>sattāsī</i>	eighty seven
<i>aṣṭāśīti</i>	<i>aṭṭhāsī</i>	eighty eight
<i>navāśīti</i>	<i>navāsī</i>	eighty nine
<i>(ekonānavati, ūnānavati)</i>		
<i>navati</i>	<i>nabbe</i>	ninty
<i>ekānavati</i>	<i>ekānabe</i>	ninty one
<i>dvi (dvā) navati</i>	<i>bānabe</i>	ninty two
<i>trayo (tri) navati</i>	<i>tirānabe</i>	ninty three
<i>chaturnavati</i>	<i>chaurānabe</i>	ninty four
<i>pañchanavati</i>	<i>pañchānabe</i>	ninty five
<i>ṣaṇṇavati</i>	<i>chhānabe</i>	ninty six
<i>saptānavati</i>	<i>sattānabe</i>	ninty seven
<i>aṣṭā (aṣṭa) navati</i>	<i>aṭṭhānabe</i>	ninty eight
<i>navānavati</i>	<i>ninānabe</i>	ninty nine
<i>(ekonāśatam, ūnāśatam)</i>		
<i>śatam</i>	<i>sau</i>	hundred

Note-For 20 there is a special word *koḍī* also. In Hindi, this is used generally for counting of cloths by washermen, for fruits etc. In Oriya and some other languages, this is the general word for 20. This is derived from the sanskrit word *koṭī* which means

limit. limit of world is 10^7 times our size, so it means 10^7 . For human body as a world, its limits are 20 fingers, so *koḍi* means 20.

6. Words for large numbers-As given in example of *Lalita-vistara*, large numbers are named in multiples of 100. Though, there are names for every multiple of ten, in practice only names in steps of hundreds were used. This is clear from number system of *Āryabhaṭa-1*, where 2 places in decimal system has been taken together. After hundred, hindi numbers are named *hazāra* (1000, thousand), *lākha* (10^5), *karoḍa* (*koṭi*= 10^7), *araba* (*arbuda*, *nyarbuda*, 10^9), *kharaba* (*kharva*, *nikharva*, 10^{11}), *nīla* (*śaṅku*, size of dark blue sky called *nīla*, 10^{13}), *padma* (10^{15} , size of creative field of galaxy), *śaṅkha* (*parārdha*, 10^{17}). First 3 places from right indicate hundreds and two-digit number. For example-

982 is nine hundred eighty-two.

In Hindi, similar name is used-*nau sau bayāsī*.

For larger numbers, places to the left are kept in groups of two and are described from the largest number, e.g.-

23, 54, 72, 85, 703 is

Teīsa araba, chauvana karoḍa, bahattara lākha, pachāsī hazāra, sāta sau tīna.

In european system, the same number will be kept in groups of 3 digits- 23, 547, 285, 703 and will be written as-

23 billion, five hundred forty seven millions, two hundred eighty five thousands, seven hundred three.

In USA and France, the large numbers are-
million= 10^6 , bilion= 10^9 , trillion= 10^{12} , quadrillion= 10^{15} ,

quintillion= 10^{18} .

In Britain and other European countries, the numbers increased in ratios of 10^6 . Here, million = 10^6 , billion = 10^{12} , trillion = 10^{18} . However, the first system is now followed everywhere.

For numbers in scientific measurements the following prefixes have been agreed for SI (International system, French abbreviation) units-

Factor	Prefix	Symbol	Factor	Prefix	Symbol
10^{24}	yotta-	Y	10^{-24}	yocto-	y
10^{21}	zetta-	Z	10^{-21}	zepto-	z
10^{18}	exa-	E	10^{-18}	atto-	a
10^{15}	peta-	P	10^{-15}	femto-	f
10^{12}	tera-	T	10^{-12}	pico-	p
10^9	giga-	G	10^{-9}	nano-	n
10^6	mega-	M	10^{-6}	micro-	μ
10^3	kilo-	k	10^{-3}	milli-	m
10^2	hecto-	h	10^{-2}	centi-	c
10^1	deka-	da	10^{-1}	deci-	d

7. Symbols for digits-There are various assumptions about shapes of number symbols. In all languages, symbol for one is one vertical line. The next numbers-2, 3, 4, etc. are combination of this line. Another view is that digits from 1 to 9 are 9 treasures of *Kubera*, lord of wealth-*Kunda*, *mukunda*, *nīla*, *kachchhapa* (tortoise), *makara* (crocodile), *kharva* (small lotus), *padma* (larger lotus), *mahāpadma* (big lotus), *śaṅkha* (conch). The picture of shapes are given on next page as per introduction in the commentary on *Grahalāghava* by *Paṇḍita Kedāradatta Jośī*.

He has also indicated shape of hindi digits as combination of single lines. Even English number forms also can be interpreted like this. For signs on electronic clocks, all numbers are made from 8 which is 2 squares joined one below the other. All other numbers are made by removing some lines from it.

La is 9th letter in line of *Ya*, *lr* is 9th vowel, so both these letters and number 9 have similar shape in Oriya and Bangla. Number 5 is of same shape as letter *pa* at start of 5th group in Hindi, Bangla, Oriya etc. 6 and 7 are called *ṣaṭ* and *sapta* as *ṣa* and *sa* are 6th and 7th letters in *ya*-group. Letter and word symbols of numerals will be described in next chapter. *Tantrāloka* has indicated shape of two letters-*E* (pronounced as english A) is triangular and *H* has two loops. Thus capital A and hindi *E* are triangular. H in english, arabic (*do-chashmi he* = *he* of double eyes) and in Hindi-all are with double loop. Since it is 8th letter in line of *ya*, 8 in many languages has double loop. Hindi A is like 3 with some appendage.










त्रिकोणमेकादशमं वह्निगेहं च योनिकम् ।

शृङ्गाटं चैव मेकारं नामभिः परिकीर्तितम् ॥

(तन्त्रालोक ३/९४, जयरथ व्याख्या)

त्रयस्त्रिंशो व्यञ्जनं द्विकुब्जः स्पर्श एव च । (निघण्टु-यास्क का नहीं, कोई अन्य पूर्ववर्ती निघण्टु) -तन्त्रालोक ३/१४२ के बाद की टीका ।

The 11th letter (*E*) is triangular and is place of fire. It may also be head with horns and is called *ekāra*. The 33rd consonant (*Ha*) is of two loops.

कुन्द	 = १'
मुकुन्द	 = २'
नील	 = ३'
कच्छप (कछुआ)	 = ४'
मकर का रूप	 = ५'
खर्व [छोटा कमल]	 = ६'
पद्म [कुछ बड़ा कमल]	 = ७'
महापद्म [सबसे बड़ा कमल]	 = ८'
शंख	 = ९'

१-	1
२=	7
३-	5
४-	4
५-	5
६-	5
७-	7
८-	5
९-	5

Chapter 3

Numbers by letters and words

1. Introduction-All over the world, numbers were represented by letters and words. Scheme of letters is called alphabet as alpha and beta are the first two letters of Hebrew and Greek schemes. For same reasons, expressing numbers by letters is called alpha-numeric system. It also means combination of letters and number symbols. These were in use all over the world and may be the reason of particular sequence of letters in the alphabets. *Kaṭapayādi* system of sanskrit was used in ancient Egypt also, and the names of planets in ancient Egypt indicate their distance in *Dhāma-jojanas* ($1/2^0$ of earth circumference=55.5 kilo meter). *Akśara-dhāma* is exponential scale has already been described in 3(1) of chapter 2. Earth measure is in linear scale. *Uṣā* (twilight) travels upto 30 *dhāmas* in west direction (of god *Varuṇa*) from place of sun rise.

सदृशीरद्य सदृशीरिदु श्वो दीर्घ सचन्ते वरुणस्य धाम ।

अनवद्यास्त्रिंशतं योजनान्येकैका क्रतुं परियन्ति सद्यः । (ऋक् १/१२३/४)

In India, twilight zone is considered upto 15^0 west of sun-rise. Thus 30 *dhāmas* are 15^0 . These vary as per latitude. For equator, $1/20$ is $40000/720 = 55.5$ Kms.- this can be taken as *dhāma-yojana*. To make it distinct from *akśara-dhāma* in exponential scale, this has been called *kśara-dhāma*. All other names in Egypt have vanished, name of moon-*Mahatāba* still remains in Persian and Arabic. In *Kaṭpayādi* system this means 5863, so mean distance of moon is $5863 \times 55.5 = 325396.5$ kms.(modern value is 382000 kms).

2. Indian use of foreign systems- For some purposes, even those names were used in India which are now thought to be Arabic. *Bauddha* literature tells that two *gurus* of *Buddha* were called *Bheruṇḍa-Kalāma* and *Ārāda-kalāma* (=saint of *Arrah* in Bihar). At present *Kalāma* word is being used only in Arabic to indicated teacher or an elevated soul-e.g. name of a famous leader was *Maulana Abul Kalāma Azad*. Prophet Mohammed also was called *Kalāma* (family name of astronomers). Writing instrument is called *kalam* (=pen) and the opening lines of Holy Koran are called *Kalamā*. The class-mates of *Buddha* were caled *Abus* (similar to word *Āpa* for respect). To understand the logic of these words, we have to see the numerical code of old script of Arabic. Its use in India shows that many systems were in use for different purposes.

Arabic codes- (Abajad) (Havvaz) (Hutti)

Arabic letters a b j d h v z hu to ye

Numerals 1 2 3 4 5 6 7 8 9 10

English sequence a b c d h i j

Codes (Kalaman) (Sa-Afas)

Arabic letters ka la ma na sa a fa sa

Numerals 20 30 40 50 60 70 80 90

English sequence k l m n

Codes (Quraśata) (Sakhaz) (jajjag)

Arabic letters Qu ra śa ta Śa kha Z ja jja ga

Numerals (100,200,300,400) (500,600,700) (800, 900, 1000)

English sequence q r s t x z

This can be seen in *Urdu-Hindi Dictionary of Mustafa Maddaha* by *Hindi Sansthan*, Lucknow-It has given many

equivalents of vedic terms used in Iraq etc. The code has also been explained in *Arabī-Śikṣā-Śāstram* (in sanskrit) of *Śrī Jagadīśa Āchārya*, by *Ramlal Kapur Trust*, *Bahalgarh-131021*, *Sonipat (Haryana)*.

From this code it is seen that some of the sequences in Roman alphabet are according to this code-e.g.-abcd, hij, klmn, qrst, xz.

For common man having elementary knowledge, it is called that he knows abc. Thus friends were called *Abus* (knowing abc). Lower persons have only 2 letters = *abe* - which is short form of aliph, Be (-th). This has become alpha, beta in Greek and A, B in English. Thus 'abe' is derogatory word. Person in the range of 10's is 'kalaman' or 'Kalāma'-thus the *gurus* of *Buddha* were *Kalāma*.

3. Kaṭapayādi-In this system, counting from 1 to 9 and 0 starts from - *ka*, *ṭ*, *pa*, and *ya*- so it is called *kaṭapayādi*. (*Ādi*=start). Indicators of numbers are-

२	२	३	४	५	६	७	८	९	०
क	ख	ग	घ	ङ	च	छ	ज	झ	ञ
ट	ठ	ड	ढ	ण	त	थ	द	ध	न
प	फ	ब	भ	म					
य	र	ल	व	श	ष	स	ह	ळ	

The rules for system are quoted below-

नञावचश्च शून्यानि संख्याः कटपयादयः ।

मिश्रे तूपान्तहल् संख्या न च चिन्त्यो हलः स्वरः ॥

(शङ्करवर्मन् -सद्रत्नमाला, ३/४)

n, *ñ* and the vowels (when standing alone) denote zero. (The consonants) beginning with *ka*, *ṭa*, *pa* and *ya* denote, in order, the digits. In a conjoint consonant, only the last consonant counts. The vowel suffixed to a consonant, too, is to be ignored, (the digits being written from right to left to form the number)

कटपयवर्गभवैरिह पिण्डान्त्यैरक्षरैरङ्काः ।

ने जे शून्यं ज्ञेयं, तथा स्वरे, केवले कथिते ॥ (अज्ञात)

The ten digits are denoted by the letters in the groups (of ten each) beginning with *ka*, *ṭa*, *pa*, and *ya*, the end letters alone being taken in the case of conjunct syllables. *Na* and *ñā* are to be understood as zero; so also the vowels when standing alone.

रूपात् कटपयपूर्वा वर्णा वर्णक्रमाद् भवन्त्यङ्काः ।

अनौ शून्यं प्रथमार्थे आ छेदे ऐ तृतीयार्थे ॥ २ ॥

(आर्यभट्ट द्वितीय-महासिद्धान्त १/२)

The consonants starting from *ka*, *ṭa*, *pa*, *ya* represent the numerals from 1 (*rūpa*) (in succession) in the order of the (respective) consonants; *ñā* and *na* denote zero. (The chronograms, when) separated (from each other) have *ā* and *ai* (at their end) in the nominative (plural) and in the instrumental (plural, respectively).

अङ्कानां वामतो गतिः ।

(Place value of) digits (of a number) move to left (in multiples

of ten).

Vedic examples-Some authors have given numerical meanings to verses of *Rgveda* by astronomers such as *Dīrghatamas* like-*Asya-vāmīya sūkta* (*Rgveda* 1/164/8) on basis of *kaṭapayādi* system. But there are too many assumptions and the conclusions are doubtful. Number system up to *parārdha* has clearly been mentioned and there are indications of word numerals also. It is difficult to find as to which of the 3 systems have been used.

This system has been very popular in *Kerala*. *Vararuchi* formed *Chandra-vākyāni* (moon-chronograms) to calculate position of moon in cycle of 248 days- see Indian Astronomy, a source book-By B. V. Subbarayapa and K. V. Sarma 13.7.14, Most famous example is value of π upto 31 places of decimal given in a verse quoted by *Svāmī Bhāratī Kṛṣṇa Tīrtha ji*, *Śaṅkarāchārya* of *Govardhana Pīṭha*, *Puri* (1880-1960) in his book-Vedic Mathematics 1965, page 362, quoted in his other book Vedic Metaphysics, 1978, page 165. The letters indicate numbers after decimal from left to right. The verse is a praise to *Śrī Kṛṣṇa*, by another combinatin of words it will be praise to *Śiva*.

गोपी भाग्य मधुव्रात शृङ्गिशोदधिसन्धिग ।

3 1 4 1 5 9 2 6 5 3 5 8 9 7 9 3

खलजीवित खाताव गलहालारसन्धर ॥

2 3 8 4 6 2 6 4 3 3 8 3 2 7 9 2

$\pi = 3.141592653589793$

2384626433832792 (31 places of decimal)

The second book also gives expression of 1/17 which also describes bad acts of *Kaṁsa*-

कंसे क्षामदाहखलैर्मलैः i.e. $1/17=0.05882353$ (approx)

17 0 0 5 882 353

Nilakaṇṭha Somayājī has indicated dates of start and completion of his work *Tantra-saṁgraha* by verses which are praise to *Viṣṇu*. In *Kaṭapayādi* notation these indicates day-numbers from start of *Kali*.

हे विष्णो निहितं कृत्स्नं जगत् त्वय्येव कारणे ।

ज्योतिषां ज्योतिषे तस्मै नमो नारायणाय ते ॥ (१/१)

गोलः कालः क्रिया चेति द्योत्यतेऽत्र मया स्फुटम् ।

लक्ष्मीशनिहितध्यानैरिष्टं सर्वं हि लभ्यते ॥ (८/३९-४०)

These two *kali* dates-16,80,548 and 16,80,553 work out to *Kali* year 4601, *Mīna* 26 and 4602, *Meṣa* 1-both dates occurring in AD 1500.

In his book-*Siddhānta-Darpaṇa*, verse 18 and in his own commentary, he has given *Kali* year and date of birth and day-count from *Kali*. Date of birth is-16,60,181-i.e.in AD 1444-(त्यजाम्यज्ञतां तर्कैः)

Dates of birth and death of *Vararuchi*, the author of *Chandra-vākyas* have been given by his son *Meḷattoḥ Agnihotri* as *Yajñasthānam surakśyam* (12,70,701) and *purudhīh samāśrayaḥ* (12,57,921) which are in AD 343 and 378.

Sūryadeva Yajvan has given his year of birth as *viśveśa* (1113) *śaka* in his commentary on *Laghumānasa* of *Munḍāla*. Another astronomer *Govinda Bhaṭṭa* was born on day *rakśed govindam arkaḥ* (15,84,362) in AD 1237 and expired on *kāliṇḍī*

priyatuṣṭaḥ (16,12,891) in AD 1295. *Achyuta Piśāraṭi* expired on *vidyātmā svar asarpat* (17,24,514) in AD 1621. *Karaṇa-paddhati* of *Putumana Somayājī* was completed on day given by *gaṇitam etad samyak* (17,65,653) in AD 1732.

Nāgārjuna has given magic square in this notation in his book (*Skanda*) *Kakṣapuṭa*.

	१ अर्क		८ इन्दु
	९ निधा		२ नारी
६ तेन		३ लग्न	
४ विना		७ -सनं	

	१		८
	९		२
६		३	
४		७	

Here, in the first line-*a* and *i* are vowels = 0, so their columns are kept vacant and they have been written in next cells. Similarly, *na* in next cells is 0, and they have been written in neighbouring cells to make the word complete.

2 1	1	1 8	8
1 7	9	2 0	2
6	1 6	3	2 3
4	2 2	7	1 5

The completed magic square is given before with sum 48 for each row and column. The general formula for magic sum $2m$ is given below-

m -3	1	m -6	8
m -7	9	m -4	2
6	m -8	3	m -1
4	m -2	7	m -9

Another magic square with sum of 100 is based on a verse given in squares and remaining part after that-

नीलं	चापी	दया	चलो
नट	भुवं	खरी	वरं
रागिणं ।	भूपो	नारी	वगो
जरा	चर	निभं	तानं

शतं योजयेत् ।। Magic Sum=100.

By placing the numbes from right to left, the square will be with sum=100.

30	16	18	36
10	44	22	24
32	14	20	34
28	26	40	06

Other examples-

ब्रह्मोत्सवः- व स म र षण्मुखः-ख म ष
4 7 5 2 2 5 6

A number 13,17, 416 is expressed as

6 1 4 7 1 3 1 6 1 4
स्तुत्यो भूस्थैः कालकृत् ; स्तु= स्+त्+उ , त्यो=त्+य्+ओ, भू=भ्+ऊ,
7 1 3 1

स्थैः=स्+थ्+ऐ, का=क्+आ, ल=ल्+अ, कृ=क्+ऋ

3438, Radian measure in minutes is expressed as-

8 3 4 3 8 3 4 3

जलेवलं; ज=ज्+अ, ले=ल्+ए, व=व्+अ, लं=ल्+अ

4. *Āryabhaṭa* notations-*Āryabhaṭa*-1 chose a system to represent numbers of up to 18 digits in two steps-

(1) He used 9 vowels of sanskrit to indicate 2 places in decimal systemby each vowel. Here, short and long vowels are taken as same as in *Māheśvara-sūtra* which is basis of grammer and tantra representing stages of creation.

अइउण् । ऋलृक् । एओङ् । ऐऔच् ।

Compared to that, the vowels normally written in sanskrit alphabet (*Devanāgarī*) are-

अ आ इ ई उ ऊ ऋ ॠ लृ लृ ए ऐ ओ औ, अं अः ।

(2) Numbers of 2 digits from 1 to 99 are represented by consonants, 1 to 25 by letters written in 5x5 square called *vargākṣara* (=square letters) from k to m.

k kh g gh ṅ 1 2 3 4 5
 ch chh j jh ñ 6 7 8 9 10
 ṭ ṭh ḍ ḍh ṇ 11 12 13 14 15
 t th d dh n 16 17 18 19 20
 p ph b bh m 21 22 23 24 25

Multiples of ten from 30 to 90 are indicated by 8 letters after square (*avarga*) from y to s. 100 can be indicated by h.

y r l v ś ṣ s h
 30 40 50 60 70 80 90 100

The whole scheme is described in a single verse only-
 वर्गक्षराणि वर्गेऽवर्गेऽवर्गक्षराणि कात् इमो यः ।

खद्विनवके स्वरा नव वर्गेऽवर्गे नवान्त्य वर्गे वा ॥२॥

The square (*varga*, वर्ग) letters (*k* to *m*) are to be written in square places (of even powers of ten). The non-square (*avarga*, अवर्ग) letters (*y* to *h*) in the *avarga* places (of odd powers of ten). The numerical value of initial *avarga* letter y is 30, because

$$य = य \times अ = 3 \times 10 = इ + म् = 5 + 25$$

Each of the nine vowels (*svaras*) have two zeros to denote place values (in powers of ten) so that *varga* letters occupy the places of even powers of ten and *avarga* letters occupy places of odd powers of ten. Two consonants together are added, consonant with vowel is multiplied.

consonant + consonant = addition

consonant + vowel = multiplication.

The chart with sanskrit forms is given here-

वर्ग	क्	ख्	ग्	घ्	ङ्	च्	छ्	ज्
	1	2	3	4	5	6	7	8
	झ्	ञ्	ट्	ठ्	ड्	ढ्	ण्	त्
	9	10	11	12	13	14	15	16
थ्	द्	ध्	न्	प्	फ्	ब्	भ्	म्
17	18	19	20	21	22	23	24	25

अवर्ग	य्	र	ल्	व्	श्	ष्	स्	ह्
	3	4	5	6	7	8	9	10

स्वर	अ	इ	उ	ऋ	लृ	ए	ओ	ऐ	औ
वर्ग	$10^0=1$	10^2	10^4	10^6	10^8	10^{10}	10^{12}	10^{14}	10^{16}
अवर्ग	10^1	10^3	10^5	10^7	10^9	10^{11}	10^{13}	10^{15}	10^{17}

Examples of numbers of revolutions of planets in a yuga of

43,20,000 years-

युगरवि भगणाः ख्युघृ, शशि चयगियिडुशुछृलृ, कुडशिबुणलृखृ प्राक् ।

शनि दुङ्ग्विध्व, गुरु खिच्युभ, कुज भदलिङ्गनुखृ, भृगुबुधसौराः ॥३॥

चन्द्रोच्च जृष्णिध, बुध सुगुशितृन, भृगु जषबिखुछृ, शेषार्काः ।

बुफिनच पातविलोभा, बुधाह्यजार्धोदयाञ्च लङ्कायाम् ॥४॥

(आर्यभट-१, आर्यभटीय १/३-४)

In a *yuga* (aeon), the eastward revolutions of the Sun are 43, 20, 000; of the Moon, 5, 77, 53, 336, of the Earth, 1, 58, 22, 37, 500; of Saturn, 1, 46, 564; of Jupiter, 3, 64, 224; of Mars, 22, 96, 824; of Mercury and Venus, the same as those of the Sun; of the Moon apogee, 4, 88, 219. Of the *śighrochchas* of Mercury, 1, 79, 37, 020; of Venus, 70, 22, 388, of other planets the same as those of the Sun. Of ascending node of Moon in opposite direction (i.e. westward), 2, 32, 226. These revolutions started at the beginning of the sign Aries on Wednesday at sunrise at *Lañkā* (when it was the commencement of the current *yuga*).

Explanation-

Sun (रवि)- ख्युघृ = 43,20,000

Vowels	ऋ	उ	इ	अ
Zero pairs	0 0	0 0	0 0	0 0
Consonants	घ	य+ख	- -	- -
Value	4	30+2	0 0	0 0

=4320000, or, 43, 20, 000

Alternately, ख्युघृ = (ख × उ) + (य × उ) + (घ × ऋ)
 = (2 × 10⁴) + (3 × 10⁵) + (4 × 10⁶)
 = 43, 20, 000

Moon (चन्द्र)-चयगियिडुशुछृलृ = 5, 77, 53, 336

Vowels	ऋ	उ	इ	अ
Zero pairs	0 0	0 0	0 0	0 0
Consonants	ल + छ	श + ङ	य + ग	य + च
Value	50 + 7	70 + 5	30 + 3	30 + 6
	57	75	33	36

= 5, 77, 53, 336 (Revolutions in a *yuga*)

Alternately, चयगियिडुशुछृलृ

= (च × अ) + (य × अ) + (ग × इ) + (य × इ) + (ङ × उ) + (श × उ) + (छ × ऋ) + (ल × ऋ)
 = (6 × 10⁰) + (3 × 10¹) + (3 × 10²) + (3 × 10³) + (5 × 10⁴) + (7 × 10⁵) + (7 × 10⁶) + (5 × 10⁷)
 = 5, 77, 53, 336.

5. Some early uses- *Āryabhaṭa-1* is credited with this system of numeration. But it appears to be in use for some purposes-may be in grammar and *tantra*. It has adopted the vowel system as in *Māheśvara-sūtras* which is basis of both. There is an ancient verse about origin of these *sūtras*, which tells that *Maheśvara* (*Śiva*) sounded his *Ḍamarū* (short drum of double cone) 14 times. This created 14 *sūtras*-

नृत्यावसाने नटराजराजः ननाद ढक्कां नव पञ्च वारम् ।

उद्धर्तु कामः सनकादिसिद्धानेतद्विमर्शे शिवसूत्र जालम् ॥

The word for *Ḍamarū* is *Ḍhakkā* as *Ḍha* = 14 in system of *Āryabhaṭa-1*. Similarly, *Bha* = 24, so *nakṣatra* names are 24- 3 of them have *pūrva* and *uttara* parts-making a total of 27. Thus, *Bhū* is world made with 24 elements of *Prakṛti* in *Sāṅkhya*, it is complete with 25 th (letter *ma*) element *Puruṣa*, so it is called *Bhūmi*. L=50, so in Roman numerals also L=50. H=100 in *Aryabhaṭa* system, though it is not used because only 2 digit

numbers up to 99 are needed for two places of each vowel. But Hundred (*Ha*) means 100 in English.

Mahābhārata has been called *Jaya* meaning 18 in *kaṭapayādi* system, because it has 18 parts and the war lasted for 18 days. The armies on both sides totalled 18 *akṣauhiṇī*, the central essence *Bhāgavad-Gītā* in it has 18 chapters. *Gītā* has explained signs of *sthitaprajña* (calm) in 18 verses. Number of *Purāṇas* and systems of learning are also 18.

Gha=4, which indicates 4 *puruṣārtha* (aims of human life)-*dharma* (right path), *artha* (wealth), *kāma* (desires, sex), *mokṣa* (release from bonds). Not doing them is *Agha*=sin. The person avoiding this sin is *Anagha*, which has been used to address *Arjuna* in *Gītā* many times.

Sound has 4 steps, so it is called *vāk* as *va*=4. Its 3 steps are in *guhā* (cave of brain) which are called *go* (*ga*=3) or *Gaurī vāk*. *Vedas* (from *va*=4) also are 4.

चत्वारि वाक् परिमिता पदानि तानि विदुर्ब्राह्मणा ये मनीषिणः ।

गुहा त्रीणि निहिता नेङ्गयन्ति तुरीया वाचो मनुष्या वदन्ति ॥

(ऋक् १/१६४/४५)

Four classes of letters or men-both are called *varṇa*, here *va*=4. *Ṣa*=6, *sa*=7, so 6 is *ṣaṭ* and 7 is *sapta*.

6. Word Numerals-Since long words have been used to indicate numbers. Debate of sage *Aṣṭāvakra* with *Kahoḍa* in court of King *Janaka* of *Mithilā* started with numbers only-what are 1, 2, 3 etc. (*Mahābhārata*, *āraṇyaka parva*, chap.134). This was called *kumāra-praśna* (quiz for children). Children were explained the concept of numbers by such examples. At many

places in vedas words indicate numbers. Like *Aśvī* (*Aśvinīkumāra* twins) always means 2. *Taittirīya Brāhmaṇa* (1/5/11/1) has used the word *Kṛta* for 4. *Svasti-pāṭha* also tells-
अग्नि जिह्वा मनवः । (ऋक् १/८९/६)

This can have two meanings-

(1) Tongues of *Agni* are 14 (= *Manu*).

(2) Fluctuations of mind are the tongues of *Agni*.

However, *Muṇḍakopaniṣad* tells that there are 7 tongues of *Agni*-

काली कराली च मनोजवा च सुलोहिता या च सुधूम्रवर्णा ।

स्फुलिङ्गिनी विश्वरुची च देवी, लेलायमाना इति सप्तजिह्वाः (१/२/४)

Agni has 7 tongues for intake-*kālī*, *karālī*, *manojavā*, *sulohitā*, *sudhūmravarṇā*, *sphulinginī*, *viśvaruchī*. (*Muṇḍaka upa.*)

सप्तप्राणाः प्रभवन्ति तस्मात् सप्तार्चिषः समिधः सप्त होमाः ।

सप्त इमे लोका येषु चरन्ति प्राणा गुहाशया निहिताः सप्त सप्त (२/१/८)

From these 7 tongues are created-7 *prāṇa*, 7 *archi* (flame), 7 *samidhā* (firewood, implement), 7 *homa* (consumption), 7 *loka*, 7-7 cave spaces.

Thus the 14 tongues of *Agni* are-7 for intake and 7 for discharge. Here, *Agni* is matter or energy enclosed in a boundary or condensed. Thus, earth within a spherical boundary is *agni*. Collection of heat energy within a closure is also *agni* (=fire). Opposite to that, *soma* is matter or energy dispersed. *Soma* is dispersed like water, dense energy of sun disperses and becomes cool near moon, so moon (its region) is *soma*.

Above, it has been told that 7 tongues of *agni* create 7 *prāṇas*. But in *Śvetāśvatara upaniṣad* (1/5) and elsewhere, it is always stated that *prāṇas* are 5. Two of the source *prāṇas* are *asat* (too

fine to perceive)-e.g. *Ṛṣi* has been called *asat-prāṇa* in *Śatapatha-brāhmaṇa* (6/1/1/1). *Pitara prāṇa* created from that (*Manusmṛti*, 3/201) are also *asat*. Further creations are *sat*.

Now, words for different numbers are indicated below-

(1) 0-Vacant space indicates zero. Similarly addition, multiplication of zero does not change it, so it can be called *pūrṇa* (complete) also. Thus, *pūrṇa*, *ākāśa* (sky) and its synonyms indicate zero-

पूर्णमदः पूर्णमिदं पूर्णात् पूर्णमुदच्यते ।

पूर्णस्य पूर्णमादाय पूर्णमेवावशिष्यते ॥ (ईशावास्योपनिषद्, मङ्गलाचरण)

That is *pūrṇa* (complete), this is *pūrṇa*, from *pūrṇa* is created *pūrṇa*. On deducting *pūrṇa* from *pūrṇa*, only *pūrṇa* remains.

It may be noted that *Antarikśa* (intermediate *lokas* or space) is not vacant. Since it has matter and energy having turbulence, we pray for its *śānti* (calmness). Similarly, *Dyau* is also having rare dispersed energy and is not vacuum.

द्यौः शान्तिः, अन्तरिक्षं शान्तिः .. (यजु. ३६/१७)

The synonyms of sky are-*Abhra*, *ambara*, *ākāśa*, *kha*, *gagana*, *nabha*, *viyat*, *vihāyasa*, *nāka*, *vyoma*.

Zero-*Śūnya*, *vindu*.

Full-*Pūrṇa*.

(2) 1-*Eka* and synonyms of earth, moon. Sun is 1 but it is felt in 12 ways in 12 signs of zodiac called 12 *ādityas*. So, sun does not indicate 1, it means 12.

Earth-*Dharā*, *dharitrī*, *dharanī*, *prthvī*, *prthivī*, *kśmā*, *kṣiti*, *bhū*, *bhūmi*, *achalā*, *vasudhā*, *vasumatī*, *ku*, *avani*, *medinī*, *mahī*, *urvī*, *rasā*, *ratnagarbhā* etc.

Moon-*Chandramā*, *chandra*, *himāṃśu*, *śaśi*, *śaśāṅka*, *śaśadhara*,

indu, *niśāpati*, *niśākara*, *niśānātha*, *kṣapākara*, *rajanikara*, *rajanīśa*, *nakṣatreśa*, *uḍupa*, *śītakiraṇa*, *sudhāmayūkha*, *dvijarāja*, *oṣadhiśa*, *abja*, *jaivātrika*, *soma*, *glau*, *mṛgāṅka*, *kalānidhi*, *vidhu*, *nakṣatreśa*, ...etc.

Others-*Rūpa* (form), *eka*.

(3) 2-*Dvi* and words for the following-

Aśvinīkumara (twins)-*aśvi*, *nāsatya*, *dasra*, *āśvineyau*, *ubhau*. Double-*yugma*, *yugala*, *yamala* (twin), *yama*, *dve*, *ubhau*, *mithuna*, *dvandva*. *Yuga* means *yugma* also, but *yuga* means 4 part *yugas*, hence it is used for 4, not for 2.

Hands-*bāhū* (singlular is *bāhu*), *hasta*, *bhuja*, *doh*. *Doh* and *bhuja* are also used for part of angle in the quadrant of 90°.

Eyes-*netra*, *akṣi*, *nayana*, *ḍṛg*, *ḍṛṣṭi*, *lochana*, *chakṣu*.

Wings-*pakṣa* (also means two halves of a lunar month), *patatra*, *pichchha*.

(4) 3-*Tri* and the words for following-

Rāma-There were 3 famous *Rāmas*-*Paraśurāma*, *Rāmachandra*, and *Balarāma*.

Agni (fire)-There are three *agnis*-*gārhapatya*, *āhavanīya* and *dakṣiṇāgni*. For 3 *agnis*, there are 3 vedas-*Ṛg* for form, *yajur* for motion and *sāma* for influence field. Synonyms are-*vahni*, *jvālā*, *śikhi*, *hutāśana*, *pāvaka*, *anala*, *dahana*, *havyavāhana*, *jātaveda*, *kṛpīṭayah*, etc.

Guṇa-These are 3-*sattva*, *rajas*, *tama*.

Loka-These are 3-*Bhūmi*, *antarikśa*, *svarga*.

Pura-Three *pura* (town) in space built by *Tāraka* were destroyed by *Śiva*- these were called *puras* of gold (*hiranya*), silver (*rūpya*, *rajata*), and iron. These are also names of the

regions of sun, moon and earth (which has iron in its core). On earth also region near pole is gold, equator is iron and middle is silver. E.g. in south America, Brazil (=iron in Hebrew), Argentina (Argentum=silver in Greek) are located.

Vikrama-Vāmana (dwaf) *Viṣṇu* had measured earth in 3 steps in *yajña* of *Asura* king *Bali*. In space, these are 3 zones of sun-heat zone upto 100 diameter, bright zone upto 1000 diameter, and *maitreya* upto 1,00,000 diameters. King has 4 policies-*sāma* (equality), *dāma* (mney), *daṇḍa* (punishment) and *bheda* (difference). *Daṇḍa* depends on *bala* (power), remaining 3 are *chhala* (bluff). Thus, *trivikrama* (3 valours) has become *tikaḍama* (*chhala*) in hindi.

(5) 4-*Chatur* and words for *vedas*, ocean, *varṇa*, *kṛta*-

Vedas-Muṇḍakopaniṣad (1/1-5) tells that *Brahmā* taught *veda* (*Brahma-vidyā* as foundation of all knowledge) to his eldest son *Atharvā*. This is called *Atharva-veda* in 2 senses-*A+Tharva* = not shaking, steady foundation, and *Atha+arva*, starting in *Arva* (Arab) where its first line about 3 sevens (786) is still used. Later on 3 more branches-*Rk*, *yajur*, *sāma* came up while retaining the original. Thus, *trayī* means 4 *vedas* and all its synonyms indicate the number 4-*Śruti*, *veda*.

Ocean-The spread of thin matter in space is called ocean. The 3 oceans of solar system, galaxy, and Inter-galactic space are *Arṇava*, *Sarasvān* and *Nabhasvān*. The ocean of uniform world is *Rasa* or *Nāra*. Parallel to that the 4 oceans on earth surface are oceans on 4 sides of Eurasia (Asia+Europe)-east, north, west and south. The separating zones between 7 continents are 7 oceans. In solar system also, the disc shaped zone up to

Uranus orbit has been called earth of 50 crore *yojanas* (1 *yojana* =1000 part of earth diameter). Within this zones covered by motion of 7 planets including moon and asteroid belt is 7 *dvīpa* (continents) and separating zone are 7 oceans. The words for ocean are-*samudra*, *abdhi*, *akūpāra*, *arṇava*, *ambhonidhi*, *jaladhi*, *ambhodhi*, *udadhi*, *sāgara*, *payodhi* etc.

Kṛta-This means *satya-yuga* having 4 parts (4 times *kali-yuga*).

Varṇa-This means letters, colour and 4 classes of society. As 4 classes, *varṇa* means 4.

(6) 5-*Pañcha* and words for arrows, airs, *prāṇa*, *bhūta*, *akśa* (organs), *Pāṇḍava* etc.

Kāmadeva (desire of sex) had arrows of 5 flowers-mango, *aśoka*, lotus, nilofer (*nīlotpala*), *mallikā* (*Amarakoṣa*). there are 5 arrows or methods of desire-*unmādana* (toxication), *tāpana* (pain), *śoṣaṇa* (taking out of fluid), *stambhana* (stopping of thought)-

अरविन्दमशोकं च चूतं च नवमल्लिका ।

नीलोत्पलं च पञ्चैते पञ्चबाणस्य सायकाः ॥

उन्मादनस्तापनश्च शोषणः स्तम्भनस्तथा ।

संमोहनश्च कामस्य पञ्च बाणाः प्रकीर्तिताः ॥

The words for arrow are-*Iṣu*, *bāṇa*, *āśuga*, *viśikha*, *mārgaṇa*, *śara*, *sāyaka*, *chitrapuṅkha*, *ropa* etc.

Prāṇa are 5 -*prāṇa*, *apāna*, *samāna*, *udāna* and *vyāna* (*Amarakoṣa* 1/1/63)

Bhūta are 5-earth, water, fire, air and sky.

Akśa or *indriya*- organs for sense and action-each are 5.

Pāṇḍava were 5 (sons of *Pāṇḍu*).

(7) 6-*Ṣaṭ* and words for *rasa* (taste-sweet, bitter, acidic, basic,

hot), *anga* (limbs of *vedas-śikṣā*, *kalpa*, *vyākaraṇa*, *chhanda*, *jyotiṣa*, *nirukta*), season, *tarka* (logic-*dravya*, *guṇa*, *karma*, *sāmānya*, *viśeṣa*, *samavāya*-in *vaiśeṣika* philosophy), *darśana* (*vedānta*, *sāṅkhya*, *yoga*, *nyāya*, *vaiśeṣika*, *mīmāṃsā*).

(8) 7-*Sapta* and words for mount, sage, notes (music), horse-Mountain-Each continent is divided into natural zones by 7 mountains called *varṣa-parvata*. Thus words of this meaning indicate 7-*aga*, *achala*, *adri*, *kṣitidhara*, *kṣitibhṛt*, *kṣmādhara*, *kṣmābhṛt*, *giri*, *naga*, *parvata*, *bhūdhara*, *bhūbhṛta*, *mahībhṛt*, *śilocchaya*, *śaila*, etc.

Sage-The 7 *Ṛṣis* were called 7 wise men in the west legends. This has 7 meanings-7 sages as source of knowledge, 7 sages starting *gotras* of human families, 7 sages of creation, 7th dimension in 10 dimensional space, stars (group of 7 stars is *saptarṣi*), *asat prāṇa*, primordial strings = *rassi* in *hindi* (strings of 10^{-35} m. size as final source of creation). The words are-*Ṛṣi*, *muni*, *tāpasa*, *yati*.

Horse-Original word is *Aśva* which means driving force. Original source of energy comes from sun as sunrays, so these are *aśva*. 7 *aśvas* of sun are thought to be 7 colours in spectrum of sunrays. But these are intensity of rays near 7 planets upto saturn including asteroid zone which have been given 7 names in *Kūrma-purāṇa* (part 1, 43/2-8), *vāyu-purāṇa* (53/44-50), *Matsya-purāṇa* (226/29-33) whose root source is *Yajurveda* (15/15-19, 17/58, 18/40). Parallel to these, 7 *vīthi*s (lane) have been formed on earth surface at 0° , and 12° , 20° , 24° -north and south latitude-the 6 zones between them are occupied in each month in north and south journeys of sun (*Atharva*

8/11/19-20, *Ṛk* 10/130/4, *Vāyu pu.* chap. 52, *Brahmāṇḍa pu.pūrva* chap. 22, *Viṣṇu pu.* part 2, chaps. 8-10 etc.) This is same as 7 lanes of sun in Book of Enoch (Ethiopian version of Bible)-chapter 4. The sea winds driving ships are also *aśva* of 7 types in 7 zones. Finally, an animal horse is also *aśva* as it pull the vehicle. It has been told that there were 1000 horses in a particular *ratha* (chariot). This is a measure of power of engine like the modern unit of horse power. Possibly, there were 7 classes of *ratha*-engines or horses according to their power. Due to 7 types, *aśva* is also called *sapti* (*sapta* = 7). Synonyms are-*aśva*, *turaga*, *turaṅga*, *ghoṭaka*, *vāji*, *haya*, *saindhava*, *sapti*, etc.

Notes in music are 7 in Indian and western music. So, *svara* means 7.

(9) 8-*Aṣṭa* and words for elephant, snake, *vasu* etc.

Elephant-*Gaja* in sanskrit has 2 meanings-elephant and measuring rod (in persian also, *gaja*=yard; english form is gauge). As measuring rod, *gaja* are 8-1000 parts of earth diameter, 32000 *angulas*, sun diameter, three units of 500, 500^2 , 500^3 , 500^4 , times sun diameter, motion of light in 1 *truṭi*. Solid surface of earth is held by 8 plates of rocks-*gaja-prṣṭha* in sanskrit and continental-plate in geology. Its synonyms are-*gaja*, *matanga*, *kuñjara*, *hasti*, *ibha*, etc.

Śiva has 8 forms as *agni* called *vasu*, 11 forms as *vāyu* called *rudras* and 12 *ādityas* as *ravi* (field of sun). Thus *vasu* = 8. In Russian, 8 is still written as *vasu*.

Boundaries of measuring rods or of continental plates are also 8- they are called snakes. The synonyms are- *sarpa*, *nāga*,

pannaga, ahi, bhujaga, bhujāṅga, pannaga, vṛtra etc.

Siddhis are also 8-*aṇimā* (becoming small), *mahimā* (big), *garimā* (heavy), *laghimā* (light), *prāpti* (reaching anywhere), *prākāmya* (getting anything), *īśitva* (managing), *vaśitva* (controlling). So, *siddhi*=8

(10) 9-is *nava* and the words for following-

Nanda-In *Mahābhārata* period, *nanda* was ninth grade of official from below. Highest was king at 10th level (*Harivaṃśa purāṇa* of *mahāpuruṣa Achyutānanda* in Oriya). In modern terms, *nanda* =secretary. The secretariate in *Rāmāyaṇa* was called *Nandigrāma* where *Bharata* stayed to manage kingdom. There were 9 kings of *Nanda* dynasty (1 father and 8 sons). So *nanda* =9.

Randhra is hole or deficit which is cause of creation of new forms and is 9th dimension. So 8 is *randhra, chhidra* etc.

There are 9 types of wealth-

पद्मोऽस्त्रियां महापद्मः शङ्खो मकर कच्छपौ ।

मुकुन्द कुन्द नीलाश्च खर्वश्च निधयो नव ॥ (शब्दार्णव)

Thus the words for treasure mean 9-*nidhi, śevadhi. Gau* (=cow) is a measure of wealth, Earlier grants by king were of 1 lakh or 1 crore *gau*. Highest coin *gau* was of gold (= made of gau), middle was *dhenu* of silver and small coin from *niṣka* (nickel, *nikka* = small in punjabi). So, *gau* =9 as wealth.

Aṅka (digits) are also 9 from 1 to 9. So, *aṅka*=9.

Graha (planets) are 9, so its words indicate 9-*graha, nabhaśchara, khechara, kheṭa* etc.

(11) 10 is *daśa* or the words for directions. *Pañkti chhanda* has 10 letters in each *pāda* (quarter), so *pañkti*=10. Direction is-

diśā, āśā, kāṣṭhā, kakubha, dik.

(12) 11 is *ekādaśa* and names of *Rudra (Śiva)-rudra, bhava, śiva, maheśvara* etc.

(13) 12 is *dvādaśa* and names of sun-*ravi, āditya, sūrya, dinakara, ina, tigmanṣu, bhāskara, dinamāṇi, arka, mārtaṇḍa, vīvasvān, taraṇi, divākara* etc.

Māsa (months) are 12.

(14) 13 is *trayodaśa* and *viśva* (=world). 13 *viśva* is mysterious. There are many assumptions- *viśvedevā* are 13 (*Amarakoṣa* tells 10 only), it is 13th word in *gaṇapāṭha* of word *sarva* (groups of words following same rule) in *Pāṇini* grammar. However, two views look plausible-(i) *Sāma* has 1000 branches, but in words, it has only 13 branches as stated in *sāma-tarpaṇa-vidhi*. In space, *sāma* is zone of influence, each *sāma* can be called a *viśva*. 1000 times earth size is equal to 13 *ahargaṇa* (measures equal to *akṣara*) i.e. 3 zones in earth and 10 outside- $1000=2^{10}$. (ii) Real reason is that there are actually 13 *viśva*. *Viśva* is any enclosure which is complete and independent; even cell (*kalila*) is a world-

अनाद्यनन्तं कलिलस्य मध्ये विश्वस्य स्रष्टारमनेकरूपम् (श्वेताश्वतर उपनिषद्)

Viśva levels bigger than man are-*bhū* (earth), *bhuvar* (4000 times bigger, *bhū-varāha*), solar system (10^7 times sun), galaxy (10^5 solar system, 10^7 of *maitreya-maṇḍala*), and whole universe. Except *bhuvar loka* or *chandra-maṇḍala* in it, others are successively 10^7 times bigger than man (*Viṣṇu-purāṇa* 1/7/5). Man is the 6th world. Below man, there are 7 worlds successively smaller by 10^5 (*Dhyānabindu upaniṣad*, 4)-*kalila* (cell), atom (*jīva*), nucleus (*kuṇḍalinī*), *jagat-kaṇa, deva-dānava,*

pīṭara, *ṛṣi*. Lord of 13 *viśva* is *Śiva*, hence *śiva-rātri* is after 13 days in dark fortnight (darkness is calm *śiva*, *rudra* is violent).

(15) 14 is *chaturdaśa* and words for-

Manu-There are 14 periods of *Manu* in each day of *Brhamā*-
काहो मनवो ढः(आर्यभट)

I.e. *aha* (day) of *Brahmā* (*Ka*) has 14 (*ḍha*) *manus*.

Alternatively, there are 14 fluctuations (*vṛtti*) of human mind-
7 tongues of intake and 7 for discharge.

In each *manu* period, there is an *Indra*, so all words for *Indra* also indicate 14-*Indra*, *śakra*, *maghavā*, *sureśa*, *sutrāmā*, *vajrī*, *vāsava*, *ākhaṇḍala*, *purandara*, *pākaśāsana*, *vṛddhaśravā*.

For 13 *viśvas*, there are 14 *bhuvana* (levels of living beings)-
1 for each *viśva* and 1 universal. Thus, *bhuvana* =14.

14 *ratna* had emerged after churning the ocean, *ratna* =14.

(16) 15 is *pañcadaśa* and words for *tithi* (15 days in a fortnight), *dina*.

(17) 16 is *ṣoḍaśa* and *kalā* (phases of moon including zero), *aṣṭi* (*chhanda* with 16 letters in each *pāda*) and words or king-
bhūpa, *nṛpa* (*soma* =moon is called *rājā* =king, it has 16 *kalā*).

(18) 17 is *saptadaśa*, *atyāṣṭi* (beyond *aṣṭi* =16, *chhanda* with 17 letters in each *pāda*). Words for cloud also indicate 17 (mostly used by *Śatānanda* in his *Bhāsvatī*). A sign can be marked on paper or cloth in 17 patterns to cover the whole plane-Wall-paper theorem (plane crystallography) in modern Algebra. Similarly, cloud covers like a sheet. So, *Puruśa* has been called 17(*Gopatha uttara* 2/13, 5/8, *Taittirīya samhita* 1/3/3/2, *Aitareya brāhmaṇa* 8/4). Words for cloud are 17-*megha*, *ghana*, *ambuda*, *vārīda*.

(19) 18 is *aṣṭādaśa* and *dhṛti* (*chhanda* with 19 letters in a *pāda*).

(20) 19 is *ekonaviṃśati*, *atidhṛti* (excess of *dhṛti*).

(21) 20 is *viṃśati*, *kṛti* (*chhanda* with 20 letters in a *pāda*), *nakha* (20 nails), *aṅguli* (20 fingers).

(22) 21 is *ekaviṃśati*, *prakṛti* (*chhanda* with 21 letters in a *pāda*), *mūrchanā* (3 groups or *grāma* in music of 7 notes each).

(23) 22 is *dvāviṃśati*, *jāti*, *ākṛti* (*chhandas* of 22 letters).

(24) 24 is *chaturviṃśati*, *jina*, *siddha* (24 *Tīrthankaras* of *Jaina*).

(25) 25 is *pañchaviṃśati*, *tattva* (25 elements of *sāṅkhya*).

(26) 27 is *saptaviṃśati*, and names for constellations (27 in number)-*nakṣatra*, *bha*, *ṛkṣa*, *tārakā*.

(27) 32 is *dvātriṃśat*, and words for teeth (32teeth of man)-
danta, *daśana*, *rada*.

(28) *Samskāra* (corrective rituals) are 48.

(29) *Tāna* (tunes in music) are 49.

7. Examples-Value of π - In *Līlāvati*-Arithmetic text of *Bhāskara*-2 its approximation up to 4 places of decimal-

व्यासे भनन्दाग्नि हते विभक्ते, खबाणसूर्यैः परिधिः स सूक्ष्मः ।

भनन्दाग्निः=अग्निः नन्दः भः ख बाणसूर्यैः=सूर्यः बाणः खः

3 9 27 12 5 0

I.e. Diameter (of a circle) multiplied by 3927 and (the product) divided by 1250 is the nearly accurate (सूक्ष्मः) circumference.

$$\pi = \frac{\text{circumference of a circle}}{\text{its diameter}} = \frac{3927}{1250} = 3.1416$$

Mādhava of *Sangamagrāma* (*Kerala*) in 14th century gave value of π up to 11 places of decimal quoted in *Kriyākramakarī* commentary on *Līlāvati*--

विबुध नेत्र गजाहि हुताशना त्रिगुणवेद भवारण बाहवः ।

नव निखर्व मिते वृत्ति विस्तरे, परिधि मानमिदं जगदुर्बुधाः ॥

That is, for a diameter of 9^{11} , the circumference is 28,27,43, 33,88,233.

Vibudha = *deva* =33, *netra* = eyes=2, *gaja* = elephant=8, *ahi* = snake =8, *hutāśana*=fire=3, *tri*=3, *guṇa* =3, *veda* =4, *bha* =stars =27, *vāraṇa* = elephant= 8. Reading from right to left, the number is 28,27,43, 33,88,233. *Nava nikharva* = 9^{11} .

The value of π from this , correct to 11 places of decimal is 3.14159265359.

Circumference of *Brahmāṇḍa* (galaxy) in *Sūrya-siddhānta* (12/82) is-

खव्योमखत्रय खसागर षट्क नाग व्योमाष्ट शून्य यमरूप नगाष्ट चन्द्राः ।

ब्रह्माण्ड सम्पुट परिभ्रमणं समन्तादभ्यन्तरा दिनकरस्य कर प्रसाराः । ।

That is, enclosure of *Brahmāṇḍa* traversed all round is the limit up to which rays of sun can reach. Its size (in *bha-yojana* = 27 *yojana* =216 kms) is 18, 71, 20, 80, 86, 40, 00, 000. The word numerals are (to be read from right to left) -*Kha* = sky = 0, *vyoma* = sky = 0, *kha-traya* =3 *khas*=3 zeros=000, *kha* =sky=0, *sāgara* = ocean=4, *ṣaṭka* =6, *nāga* = snake=8, *vyoma* = sky=0, *aṣṭa* = 8, *śūnya* = 0, *yama* =twin=2, *rūpa* =form=1, *naga* = mountains=7, *chandra* =1.

Sūryadeva Yajvan (born 1191 AD) in his commentary on *Āryabhaṭīya* explains the radian measure of angle (whose arc is equal to radius) as a problem using both word numerals and *kaṭapayādi*-

यदि र-ड-वसु-यमल-रस मितपरिधेरयुतद्वयं व्यासः,

2 3 8 2 6 20,000

ख-ख-षड्घन लिप्तात्मक परिधिश्चक्रस्य को व्यासः इति ?

0 0 $6^3=216$

That is, if 62, 832 is the circumference of a circle of diameter of 20, 000 units, what is the circumference of a circle of radius 21, 600 units ?

Sometimes, there is confusion in multiple notations as in above example, *ghana* is power of 3, but as a word numeral it can mean cloud =17 also. Similarly, *dvi-rada*, read separately means *dvi*=2 and *rada*=teeth=32, i.e. 322. Taking single word, it means elephant (having two teeth)=8.

Chapter 4

Svastika and philosophy

1. Operational words-In continuation of the previous chapter, words for various operations like addition, division, square, roots are described. It is described here, as it explains the philosophy of the operations. This is compiled in commentary of Prof *K.S. Shukla* on *Bhāskara* commentary of *Āryabhaṭīya*, appendix III.

1. Addition(+)-The terms used to denote the operation of addition are derived from the roots *as* =to move, to shine or to take (with prefix *sama*= to be united or added), *i* or *ī*= to move (with prefix *an*, *upa*, *sama*, or *saha* =to be connected, united or added), *kala* =to move, to count (with prefix *saṃ* =to add or sum up), *kśip* =to throw, to add, *chi* =to select, collect, combine (with prefix *upa* =to increase), *dā* =to give, *piṇḍ* or *piḍi* = to unite, *prch* = to touch, combine (with prefix *saṃ* =to unite or add), *miśra* =to mix, *ṛdhu* = to increase, *yu* = to mix, to separate, *yuj* (*yujir*) =to add or unite.

(i) From root *as* =to move, to shine or to take-
samasta, *samāsa*, *samāsita*.

(ii) From root *i* or *ī*= to move-
anvita, *upeta*, *samanvita*, *samaveta*, *sameta*, *sahita*.

(iii) From root *kala* =to move, to count-
saṃkalana, *saṃkalita*.

(iv) From root *kśip* =to throw, to add-
kśipa, *kśipta*, *kśiptam*, *kśiptvā*, *kśipet*, *kśipyate*, *kśipyante*, *kśepa*, *kśepyam*, *kśepyā*, *parikśipya*, *parikśipyante*, *parikśipet*, *parikśipta*, *prakśipta*, *prakśipya*, *prakśipyate*, *prakśipyante*,

prakśepa, *vinikśipet*, *saṃkśepa*.

(v) From the root *chi* =to select, collect, combine-
upachaya, *upachita*, *upachīyante*, *upachīyamāna*.

(vi) From the root *dā*=to give-
dātvā, *dātavya*, *dīyate*, *dīyante*, *deya*, *deyā*.

(vii) From the root *piṇḍ* or *piḍi* = to unite-
piṇḍita, *sampiṇḍya*.

(viii) From the root *prch* = to touch, combine-
samparka.

(ix) From the root *miśra* =to mix-
miśrita, *sammiśra*.

(x) From the root *ṛdhu* = to increase-
vardhate, *vivardhate*, *ṛddhi*.

(xi) From the root *yu* = to mix, to separate-
yuta, *yuti*, *samyuta*, *samyuti*.

(xii) From the root *yuj* (*yujir*) =to add or unite-
niyojya, *yukta*, *yuktya*, *yoga*, *yojayitavyam*, *yojayet*, *yojitā*, *yojyam*, *yojya*, *yujyate*, *yojyante*, *yojyāh*, *vinijojya*, *samyukta*, *saṃyoga*, *saṃyojita*, *saṃyojya*, *saṃyojyamāna*.

(xiii) Other terms and forms-
adhika, *ādhya* (antonym of *vihīna*), *ekīkṛta*, *kalpa* (addition), *dhana* (addition), *udaya* (addition).

2. Substraction (-)-The terms for the operation of subtraction are derived from the roots *as* = to move, to shine or to take (with prefix *apa* =to leave, throw away), *ī* = to move (with prefix *apa* =to leave, omit), *ūn* =to lessen, *r* =to give up, *kśi* =to decay or waste or diminish, *grah* =to take away, *chi* =to select, collect, combine (with prefix *apa* =to diminish), *tyaj* =to aban-

don or discard, *nī* =to guide, arrive, carry (with pefix *apa* =to diminish), *pat* =to fall, descend, *yuj* (*yujir*) = to add or unite (with prefix *vi* =to disunite), *rah* =to discard, *vr̥ñ* =to select (with prefix *vi* =to open), *vr̥jī* =to exclude, stop, *śiṣ* =to remain at last (remainder), *śudh* =to purify, *śliṣ* =to join (with prefix *vi*, to disunite), *hāk* =to leave, *hr̥ñ* = to take away.

(i) From root verb *as* =to move, to shine or to take-*apāśya*.

(ii) From root verb *ī* = to move-*apāya*.

(iii) From root verb *ūn* =to lessen-*ūna*, *ūnakam*.

(iv) From root verb *r* = to give up- *ṛṇa*.

(v) From root verb *kśi* =to decay or waste or diminish- *kśaya*.

(vi) From root verb *grah* = to take away- *pragr̥hya*.

(vii) From root verb *hi* = to select, collect, combine-
apachaya, *apachayātmaka*, *apachīyate*, *apachīyante*.

(viii) From root verb *tyaj* =to abandon or discard-
tyaktvā, *tyajet*, *tyajante*.

(ix) From root verb *nī* =to guide, arrive, carry-
apanayana, *apanayet*, *apanīte*, *apanīya*, *apanīyate*, *apanīyante*,
samapanīya.

(x) From root verb *pa t* =to fall, descend-
nīpatita, *nīpātya*, *patita*, *pātayitvā*, *pātita*, *pātayate*.

(xi) From root verb *yuj* (*yujir*) = to add or unite-
viyuktī, *viyoga*.

(xii) From root verb *rah* =to discard-
rahita, *virahita*.

(xiii) From root verb *vr̥ñ* =to select-
vivara, *vivarakam*.

(xiv) From root verb *vr̥jī* =to exclude, stop-

varjita, *vivarjita*.

(xv) From root verb *śiṣ* =to remain at last-

avaśiṣṭa, *avaśeṣa*, *viśiṣṭa*, *viśiṣyate*, *viśeṣa*, *viśeṣaṇa*, *viśeṣita*,
viśeṣyate, *śiṣṭa*, *śiṣyate*, *śeṣa*, *śeṣayet*.

(xvi) From root verb *śudh* =to purify-

pariśuddha, *pariśodhya*, *praviśuddha*, *praviśodhayet*,
praviśodhya, *viśuddha*, *viśodhayet*, *viśodhita*, *viśodhite*,
viśodhyam, *viśodhya*, *viśodhyate*, *viśodhyante*, *viśodhyā*,
śuddham, *śuddha*, *śuddhi*, *śuddhe*, *śuddhyati*, *śuddhyanti*,
śuddhyante, *śuddhyet*, *śodhita*, *śodhanam*, *śodhanīyam*,
śodhayitvā, *śodhayet*, *śodhyam*, *śodhya*, *śodhyate*, *śodhyā*,
saṁśuddha, *saṁśuddhi*.

(xvii) From root verb *śliṣ* = to join-

aviśliṣṭa, *viśliṣṭa*, *viśleṣa*, *viśleṣita*.

(xviii) From root verb *hāk* = to leave-

parihīna, *vihīna*, *hitvā*, *hīna*.

(xix) From root verb *hr̥ñ* = to take away-*hr̥āsa*.

(xx) Other relevant terms-

agra (remainder, residue), *antara* (difference).

3. Multiplication (x)-The terms denoting the operation of multiplication are derived from the root verbs-*as* =to move, to shine or to take (with prefix *abhi* =to repeat, multiply), *kśudir* =to grind, *guṇa* =to invite, multiply, *taḍa* =to beat or strike, *vr̥jī* = to exclude, *vr̥tu* =to behave or remain (with prefix *ut*), and *han* = to kll, to go, to get.

(i) From root verb *as* =to move, to shine or to take-

abhyasta, *abhyasya*, *abhyāsa*, *samabhyasta*, *samabhyasya*.

(ii) From root verb *kśudir* =to grind-

kśuṇṇa, saṁkśuṇṇa.

(iii) From root verb *guṇa* =to invite, multiply-

guṇa, guṇaka, guṇakāra, guṇanā, guṇayitvā, guṇayet, guṇita, guṇya, guṇyate, guṇyāh, guṇyāt, saṅguṇa, saṅguṇayya, saṅguṇā, saṅguṇita, saṅguṇāh, saṅguṇya.

(iv) From root verb *taḍa* =to beat or strike-
abhitāḍita, tāḍita.

(v) From root verb *vṛjī* =to exclude-*saṁvarga, vargaṇā.*

(vi) From root verb *vṛtu* =to behave or remain-*udvartanā.*

(vii) From root verb *han* =to kill, to go, to get-
abhinighna, abhihata, abhihatya, āhata, āhatya, āhanyāt, ghāta, ghna, nighna, nihata, nihatya, praṇighna, praṇihatya, viśaṁhati, saṁhati, saṁhatya, saṁhanyāt, samāhata, hata, hatam, hṛtvā, hatih, hatvā.

4. **Division (÷)**-The terms used to denote the operation of division are derived from root verbs-*khaṇḍ* (*khaḍi*) =to break, *chhid* (*chhidir*) = to break, divide, *bhaja* = to distribute, divide, *bhañja* = to break to pieces, *vṛtu* =to behave or remain (with prefix *apa*), *hṛñ* =to take away, destroy.

(i) From root verb *khaṇḍ* (*khaḍi*) =to break-*khaṇḍyāt.*

(ii) From root verb *chhid* (*chhidir*)= to break, divide-
chhitvā, chhidyate, chhidyāt, chheda, chhedyā, saṁchheda.

(iii) From root verb *bhaja* =to distribute, divide-
pravibhajet, bhakta, bhakte, bhaktavya, bhaktvā, bhajana, bhājita, bhajet, bhāga, bhāgahāra, bhāge hṛte, bhājayet, bhājita, bhājayam, bhājya, bhājyā, vibhakta, vibhajet, vibhajeta, vibhajya, vibhajyate, vibhājita, vibhājayet.

(iv) From root verb *bhañja*= to break to pieces-*bhañktvā.*

(v) From root verb *vṛtu* =to behave or remain-*apavartana.*

(vi) From root verb *hṛñ* =to take away, destroy-

apahṛta, āharet, uddhṛta, upāhara, vihṛta, saṁharet, saṁhṛtam, saṁhṛta, samāhṛta, samuddhṛta, haratu, haret, hartavya, hṛta, hṛti, hṛte, hṛtvā, hriyate, hriyamāna.

5. **Square**-Terms used-*kṛti, yāva, varga, vargitam, vargaṇā.*

6. **Square root**-Terms used-*dvigatamūla, pada, mūla, vargamūla.*

7. **Cube**-Terms used-*ghana, trigata, vṛnda, sadṛśatrayābhyāsa.*

8. **Cube root**-Terms used-*ghanamūla, trigatamūla.*

2. ***Svastika gaṇita***-Signs of numbers based on *svastika* can

| + † ‡ § ¶ · ¸ 9

1	2	3	4	5	6	7	8	9	10
	¶		¶†	¶‡		¶	¶		
	11		12	13		14			
	2	¶	3				¶	9	
20	30		90			
¶		¶		¶					
21	22	23						

be written as per the chart shown here. Alternatively,

10 =d (ḍ), 20=d2, 30 =d3 etc also can be written.

Notation based on *Tantra* can be made. The letters for 5 *mahābhūtas* are-earth-*laṁ*, water- *vam*, fire-*ram*, air- *yam*, sky-*ham*. On same lines, signs for different elements can be taken-*Aham* (self) =*am*, *maha*(collection of matter) =*mam*, *tamas*

(inertia, darkness) = *tam*, *prakṛti* (nature) = *pam*, *sarveśvara* (Supreme lord) = *sam*. Symbols of numbers in hindi are indicated below-

१ ए, २ द्वि, ३ त्रि, ४ च, ५ पन्, ६ षष्, ७ सन्, ८ अन्, ९ न,

१० द

१०० श

१००० स

१०,००० अ

१,००,००० ल

१०,००,००० प्र

१,००,००,००० क

१०,००,००,००० अर्

१,००,००,००,००० अब्

१०,००,००,००,००० ख

१,००,००,००,००,००० नि

१०,००,००,००,००,००० म

१,००,००,००,००,००,००० शं

१०,००,००,००,००,००,००० ज

१,००,००,००,००,००,००,००० अत्

१०,००,००,००,००,००,००,००० मम्

१,००, ००,००,००,००,००,००,००० प

परार्द्ध=प, द्विपरार्द्ध=द्विप, पर=पर, परतर=परत, परतम=परम,

असङ्ख्य= 卐 , अनन्त=०.

१०= द, ११=दए, १२=दद्वि, १३=दत्रि, १४=दच, १५=दपन्, १६=दषष्,

१७=दसन्, १८=दअन्, १९=दन, २०=द३, २१=द२ए, २२=द२द्वि, ३०=द३,

१०=द९, १००=श, १०००=श९, १००००=स, १०००००=स९, १००००००=अ,

परार्द्ध=प, द्विपरार्द्ध=द्विप,

परार्द्ध × परार्द्ध = पर

परार्द्ध × परार्द्ध × परार्द्ध = प३

पूर्णमदः पूर्णमिदं पूर्णात्पूर्णमुदच्यते ।

पूर्णस्य पूर्णमादाय पूर्णमेवावशिष्यते ॥

That is complete (full or all), this is all. When all is deducted from all, all remains.

Source of all numbers 1 is all. The numbers derived from that 2, 3, etc also are all. From complete, arises complete only. From the complete 1, created numbers 2, 3 .. also are complete. From any number which is complete, if itself is deducted, the remaining zero also will be complete.

Thus, zero is unchanged in operations of addition, subtraction, multiplication and division. This is not absence, it is start of numbers from 1. The formless world is zero, and forms start with 1.

0+0=0, 0-0=0, 0×0=0, 0÷0=0 (actually undefined)

Praṇava of letters is *Aum* (ॐ)

Similarly, *praṇava* of numbers is 190=start of numerals, last numeral, and end with formless 0.

3. Nature of *Svastika*-The word *Svastika* has two parts-

Sva=*Ātmā* (soul) in *vedānta*.

Sti=(i) s=*Sattva-guṇa* =existence.

(ii) t = *tamoguṇa* =inertia, darkness.

(iii) i =fourth vowel =*kāmakaḷā*, fulfilling desires, art of sex.

Kāma arises from *rajo-guṇa* (motion, action)-

सोऽकामयत् ।

He (God) expressed *kāma*.

तदैक्षत (छान्दोग्योपनिषद् ६/३)

He (God) saw or desired.

स इक्षाञ्चक्रे (प्रश्नोपनिषद् ६/३)

He (God) desired.

Thus, the 4 parts of *Praṇava* (ॐ) described in *Māṇḍūkyaopaniṣad* are same as *Puruṣa* and *Prakṛti* of 3 *guṇas*. Three letters of *Aum* are 3 *guṇas* with *Puruṣa* and abstract sound is Supreme *ātmā*.

प्रणवत्वात् प्रकृतिरिति वदन्ति ब्रह्मवादिनः । (रामोत्तरतापिनी उप.४)

That is *Praṇava* is *Prakṛti* as the knowers of *Brahma* tell.

Shape of *Svastika* has come from the verse used for *Svasti-vāchana* (uttering for well-being) which occurs in all the *vedas*.

स्वस्ति न इन्द्रो वृद्धश्रवाः स्वस्ति नः पूषा विश्ववेदाः ।

स्वस्ति नस्तार्क्ष्यो अरिष्टनेमिः स्वस्ति नो बृहस्पतिर्दधातु ॥

(ऋक् १/८९/६, साम १८७५, वा.यजु. २५/१९, तैत्तिरीय आरण्यक १/१/१, २१/३, १०/१/९)

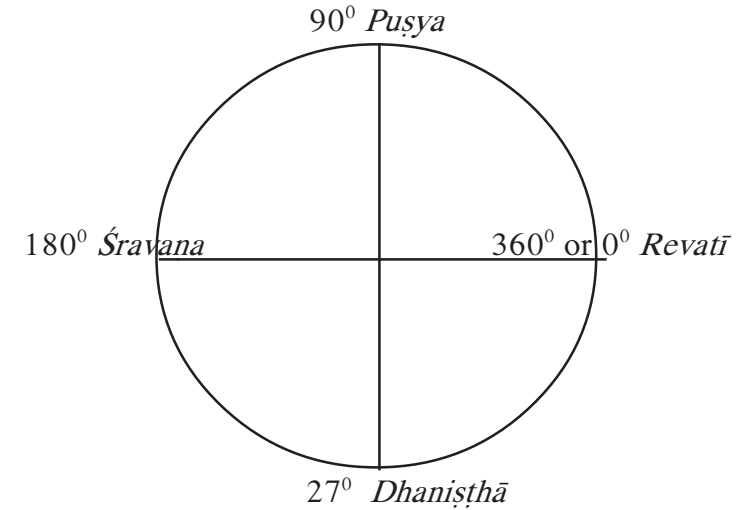
Astronomical meaning of this verse is represented by the figure of *Svastika*. As seen from north pole of earth, the zodiac moves clockwise. Such motion with 4 quadrants of a circle is the *Svastika*. The stars on 4 points of quadrant are represented by their lords as shown below-

Indra-lord of *śravaṇa*, in shape of ear-ring which is an extension of ears (*Ṛddhaśravā*, *śravā* = years, *ṛddha* = old, extension), *Pūṣā* = lord of the last star *Revatī* -thus covering full circle; or knowing *viśva* = world (*Viśvavedā*)

Tārksya = *garuḍa*, or with 3 stars (*tārksya* = *tri* + *rkśa* = 3 stars). The star *Dhaniṣṭhā* is called *tri-charaṇa*, i.e. 3 steps of *Viṣṇu*. This has 3 stars. When *Viṣṇu* in form of sun reaches this star, its northward motion starts, i.e. that marks the end of severe winter. Thus it is limit (*nemi*) of sorrow (*ariṣṭa*).

Bṛhaspati is lord of *puṣya* star which gives *puṣṭi* (nourishment), so it is prayed to maintain us (*dadhātu*).

The figure of stars in circle is shown below-



Physical meaning- *Svasti* = well being. The welfare of man is by fulfilling 4 *puruṣārthas*-*dharma*, *artha*, *kāma*, *mokṣa*. *Dharma* (righteousness) is protected or enforced by the king *Indra* who is number 1 (*ṛddha*) in hierarchy (*śravā*=line), i.e. *ṛddhaśravā*. *Artha* (wealth) gives nourishment indicated by *puṣā*. *Kāma* (Desires) are fulfilled by removing obstacles (*ariṣṭa*). Finally, *mokṣa* (emancipation) is by knowing the world (*viśvavedā*).

4. Nature of zero and one-*Puruṣa* is 1 which is of 5 types

in *vedānta*-

- (i) *Nirbīja* (without seed)-A rope is essentially same thing as any other, as all are transforms of same formless.
- (ii) *Sabīja* (with seed)-It is transformed as a rope.
- (iii) *Samśaya* (doubt)-In darkness, we may think it as rope, snake, rod etc. without definite knowledge.
- (iv) *Viparyaya* (decay, change)- The rope may become decayed and useless.
- (v) *Vilāsa* or *vivarta* (transform)- The rope may be used as a garland, or rod, knot etc.

In all these forms, *Brahma* or *Puruṣa* is 1-

दिव्यो ह्यमूर्तः पुरुषः स बाह्याभ्यन्तरो ह्यजः ।

अप्राणो ह्यमनाः शुभ्रो ह्यक्षरात्परतः परः ॥ (मुण्डकोपनिषद् २/१/२)

Radiant formless *Puruṣa* is *Aja* (unborn) in and out. This is without *prāṇa* (energy), without mind, without decay and beyond *para* (root *prakṛti*) also.

अव्यक्तात् पुरुषः परः । पुरुषान्न परं किञ्चित्सा काष्ठा सा परा गतिः ।
(कठोपनिषद् १/३/११)

Puruṣa is beyond formless (*prakṛti*) also. There is nothing beyond *Puruṣa*, He is *kāṣṭhā* (limit) and *parā gati* (final abode).

न तस्यकार्यं करणं न विद्यते न तत्समश्चाभ्यधिकश्च दृश्यते ।

(श्वेताश्वतर उपनिषद् ६/८)

एको देवः...साक्षी चेता केवलो निर्गुणश्च । (श्वेताश्वतर उपनिषद् ६/११)

He does not have work or implements, none is seen equal or greater than Him. The one *Deva* is *sākṣī* (witness), conscious and formless.

These verses show unity of form in formless space.

सदेव सोम्येदमग्र आसीदेकमेवाद्वितीयम् । .. तदैक्षत बहुस्यां प्रजायेयेति ।

(छान्दोग्य उपनिषद् ६/२/१,३)

That *Deva* was *Sat* (existant) and one only, without second. He thought-I will create many beings.

एकं बीजं बहुधा यः करोति । (श्वेताश्वतर उपनिषद् ६/१२)

Wo makes the single seed manifold.

त्वं स्त्री त्वं पुमानसि त्वं कुमार उत वा कुमारी । त्वं जीर्णो दण्डेन वज्जसि ।

(श्वेताश्वतर उपनिषद् ४/३)

You are female, male, young or maden. Or, You are old man moving with a stick. This shows unity of *Brahma* in zone of doubt.

सर्वं खल्विदं ब्रह्म । (छान्दोग्य उपनिषद् ३/५/१)

All is *Brahma* definitely.

सर्वं चिन्मात्रमेव हि । (तेजोबिन्दूपनिषद् २/४०)

All is just a point-consciousness.

सर्वमात्ममयं जगत् । ब्रह्मात्रमिदं सर्वम् । (तेजोबिन्दूपनिषद् २/४०)

The whole world is pervaded by *ātmā*. All is nothing but *Brahma*.

Five fold 1 in mathematics is-

(1) *Nirbīja* 1-0=1

(2) *Sabīja* 1+0=1, 0+1=1.

(3) Multiplication 1x1=1.

(4) Division or subtraction 2-1=1, 3-2=1

1÷1=1

(5) Increase 1+1=2, 2+1=3

or, 1+1+1=3.

Prakṛti is like 0, but it is not non-existent again as 0. It is

material, but can not do any work or creation without *Puruṣa* which is the consciousness.

मयाध्यक्षेण प्रकृतिःसूयते सचराचरम् ।

हेतुनानेन कौन्तेय जगद्विपरिवर्तते । (गीता ९/९)

Under My lordship (of *Brahma*), *Prakṛti* creates moving and non-moving. By that cause only, world is transformed.

यावत्सञ्जायते किञ्चित्सत्त्वं स्थावरजङ्गमम् ।

क्षेत्रक्षेत्रज्ञसंयोगात्तद्विद्धि भरतर्षभ । (गीता १३/२६)

Whatever is created moving or non-moving, know it by combination of *kṣetra* and *kṣetrajañā*.

मायां तु प्रकृतिं विद्यान्मायिनं तु महेश्वरम् ।

तस्यावयवभूतैस्तु व्याप्तं सर्वमिदं जगत् । (श्वेताश्वतर उपनिषद् ४/१०)

Know *māyā* as *prakṛti* and *Māyī* (lord of *māyā*) as *Maheśvara*. By his components only the whole world is pervaded.

Like *Prakṛti*, 0 also has 5 forms in mathematics-

- (1) Self balance- $0+0=0$, $0-0=0$, $0 \times 0=0$, $0 \div 0=0$.
- (2) Balance with other numbers- $0 \times 1=0$, $1 \times 0=0$, $0 \div 1=0$.
- (3) With negative number $0-1=-1$.
- (4) Negative zero- $1-0=1$, $2-0=2$.
- (5) Increasing value of numbers-10, 20, .. 100. $1 \div 0=$

Self balance is like balance of 3 *guṇas*. Balance with other numbers is like *sattva-guṇa*. With negative number, it is like *tamo-guṇa*. Negative zero is mixed *guṇa*. Increasing value of numbers is like *rajo-guṇa*.

5. Role of 108-This number is used for great men. Normal man is addressed as a *śrī* before his name. But 108 *śrī* is placed before name of *Guru* (preceptor). On this analogy,

Jagadguru (Head of *Śṅkarāchārya pīṭha*) is called 1008 *Śrī*.

Here, 1=*Brahma*. 0 is root *Prakṛti* in which all 3 *guṇas* are balanced. 8 is the number of forms of *Prakṛti* from which further creations- i.e. 16 *vikṛtis* are made. In forward sequence of 1, 0, 8 means a person knowing *Brahma*, root *prakṛti* and its transformation into 8 forms. The reverse sequence of 8, 0, 1 means the person seeing unity of 8-fold *prakṛti* and its root with *Brahma*.

Astronomically, this is distance of sun and moon in units of their diameter. That is

Distance of sun from earth =sun diameter x 108.

6. Signs of operation-Root sign is a dot or point. By its motion, sign of minus has been formed. Words for subtraction have been formed by verbs meaning-to go, to take away, etc (see para 1.2). About origin of signs minus sign (*ṛṇa*) has been called first sign and dot is its small form (*chid ṛṇa*). These signs were first devised by *Gaṇeśa*, so *Aṅkapadīyam* has been also called *Gaṇanātha-gaṇita*. First *Gaṇapati* (in human form) was called *Brahmaṇaspati*, who was *Kavi* and *Guru*. In *Gaṇapati-atharva-śīrṣa*, verse 7, *akśara* with half moon sign (*ardhendu-lasita*) has been described from him.

गणानां त्वा गणपतिं हवामहे कविं कवीनामुपमश्रवस्तम् ।

ज्येष्ठराजं ब्रह्मणा ब्रह्मणस्पत आ नः शृण्वन्नृतिभिः सीद सादनम् ॥१॥

Jyēṣṭharāja Brahmā (in human form) gave him the right to spread the knowledge of script. He made *Sāma* (song, psalm of Bible) and bound it with *chhanda* (*kavi* =divides into *kavala* =small packets) and preserved with *Rta* (writing, spread matter or sign).

विश्वेभ्यो हित्वा भुवनेभ्यस्परि त्वष्टाजनत् साम्नः कविः ।

स ऋण चिदृणया ब्रह्मणस्पति र्द्रुहो हन्तमह ऋतस्य धर्तरि ॥१७॥

(ऋक् २/२३/१, १७)

He held the *Rta* (writing, spread of signs) with *ṛṇa* (dash -, minus sign) and by *chid-ṛṇa* (minute part of dash, dot). By three signs (1 sign = 1 combination of dash+dot), signs of *Devas* were written-

देवलक्ष्मं वै त्र्यालिखिता तामुत्तर लक्ष्माण देवा उपादधत् .. (तैत्तिरीय संहिता ५/२/८/३)

Devas are *prāṇa* of 33 *dhāmas* of sun. Sign of 33 *devas* are letters (consonants) from *k* to *h*. This script is a *nagara* (city = *chiti* = arrangement) of *devas* in symbols, so it is called *Devanāgarī*. Signs of all letters are 49 *maruts* (airs of 49 zones of galaxy), so its creator was called *Marut* or *Vāyu*-

वाग्वै पराच्यव्याकृतावदत् ते देवा इन्द्रमब्रुवन्निमां नो वाचं व्याकुर्विति सोऽब्रवीद्वरं वृणै, मह्यं चैव वायवे च सह गृह्याता इति । (तैत्तिरीय संहिता ६/४/७)

Vāk (words) were in ancient times without parts. *Bṛhaspati* had given names to each word called *pada*- (Rk 10/71/1)). On request of *Devas*, *Indra* took help of *Vāyu* and broke the words (into *dhātu*, *upasarga*, *pratyaya* etc). As is was *vyākṛta* (separated into parts), it was called *vyākaraṇa* (grammar).

Each sign is combination of dash and dot-can be indicated by 0, 1. Their combination can be of $2^2 = 4$ types- 00, 01, 10, 11. Combination of 3 signs can be in $2^6 = 64$ ($4 \times 4 \times 4$) types. Thus, *Brāhmī* script has 64 letters. *Devanāgarī* script with 49 letters, three letters for *kṣetrajaṇa* (*kṣ*, *tra*, *jña*), 10 symbols of numbers

also can be written with this scheme. Actually, I-ching script of China still uses 64 combinations of 3 pairs of dash and dot. This was adopted for Morse-code for telegrams and was basis of ASCII code in Computer.

Addition is joining of two parts indicated by two parts of dash on left and right sides joined. Or, it may be taken as negative of negative, hence two dashes are joined perpendicularly.

Multiplication is repeated joining-like twisting of a rope by which its fibres are joined repeatedly-this process is called *guṇana*, so multiplication also in called *guṇā*. The original form of components is completely destroyed in *guṇana*, thus it is indicated by verbs having meanings-to kill (see para 1.3). In English also the process is called cross which means to kill, or to multiply both.

Division separates two parts. Single part is space indicated by vacant circle, whose short form is a dot (*vinḍu*). By division of a *vinḍu* in two, it becomes *visarga* (discharge, creation), in which two dots are placed one below other separated by a horizontal line.

विसर्गः कर्म संज्ञितः । (गीता ८/३)

I.e. *Visarga* is named *karma* (action).

वियोगे दोषदर्शी यः संयोगं स विसर्जयेत् ।

(महाभारत, आश्रमवासिक पर्व ३४/१५)

The person who sees fault in *vīyoga* should discard (*visarga*) seeing fault in gain and loss.

7. Classification of Mathematics-For two branches of knowledge, *Gaṇeśa* (countable) and *Sarasvatī* (*rasa* = abstract),

there are two broad classes of mathematics-

<i>Gaṇeśa</i>	<i>Sarasvatī</i>
Discrete	Field
Algebraic (<i>raśi</i>)	Geometric (<i>kṣetra</i>)
Pure	Applied
Exact (<i>Sakṛta</i>)	Approximate (<i>Asakṛta</i>)
Finite	Infinite
Theory (<i>Bhāva</i>)	Algorithms (<i>Bhāvita</i>)

Bhāskara I has mentioned *rāśi* and *kṣetra* methods of *gaṇita* in his commentary on *Āryabhaṭīya* (translation with comments by Prof. *Kripashankar Shukla*, published by Indian National Science Academy, Delhi-2 in 1955). Texts like *Siddhānta darpaṇa* give *sakṛta* (exact or one time) method and *asakṛta* (repeated, successive approximation). *Bhāskara*-I also mentions 4 *bījas*-sub classes of *rāśi*-

1st-*yāvat-tāvat*, 2nd-*varga-avarga*

3rd-*ghana-aghana*, 4th-*viśama* (mixed)

These are simple euations, quadratic, cubic, and of more than one unknown or mixed.

He mentions 4 treatises of earlier *āchāryas* at two places (A-1/1) and A-2/9)-*Maskarī*, *Pūrana*, *Mudgala* and *Pūtana*-which may indicate 4 *pāda* (quarters) of mathematics like 4 *pāda* of *Puruṣa* or *Brahma*. *Maskarī* may mean collection of algorithms (*maska* =repetitive exercise, *maskarī* = wielder of rod, log = rod in English). *Mudgala* (*mudga* = grain of pulse) may mean discrete mathematics. *Pūrana* and *Pūtana* appear to be two opposite branches of mathematics like Integral (sum of area, volume elements = *pūrana*, *Pūtana* = rectification, differential cal-

culus). *Mahābhārata*, *śānti parva* (47/12) mentions *Bhāskara* (or *Maskarī*) and *Pūtana* among the sages surrounding *Bhīṣma* at his last moment. *Maudgalya* (of *Mudgala* clan) also has been mentioned but no indication has been given about their knowledge of mathematics. *Maskarī* and *Pūrana* have been mentioned as mathematicians in *Bodhaprābhṛta* (commentary by *Śrutasāgara*) and *Bhāvaprābhṛta* of *Kunda-kunda*. Each of mathematics branches gave rise to a school of philosophy and these schools were uprooted after advent of *Buddha* (1887-1807 BC)-*Makkhali Gośāla* (*Maskarī*), *Pūrana Kāśyapa* and *Mudgala* are mentioned as three sects uprooted by him. Possibly that period ended the formal texts of mathematics as well as surgery in name of non-violence (see for example *Majjhima-Nikāya*. Detailed analysis is in *Bauddha-dharma and Bihar*, by *Sri Havaladar Tripathi*, published by *Bihar Rashtrabhasha Parishad*, Patna-4).

It is interesting to note that some of the texts are named in *Gaṇta-sūtras-Pūrana-apūraṇābhyām* (*Pūrana*), *Saṅkalana-vyavakalana*, *Vyaṣṭi-samaṣṭi* (opposite branches listed above) or *Śūnyamanyat* (other zeros). This link itself shows wider meanings of *Gaṇta-sūtras*.

8. Prayer-The prayers given as *maṅgalācharaṇa* of *Aṅka-padiyam* (*Gaṇanātha-gaṇita*) is given below-

गणेशं प्रमथाधीशं निर्गुणं सगुणं विभुम् ।

योगिनो यत्पदं यान्ति तं गौरीनन्दनं भजे ॥२॥

(गणेश पूर्वतापिन्युपनिषद्)

Gaṇeśa is lord of *Pramatha* and is both lord of *nirguṇa* (abstract) and *saguṇa* (concrete). We pray the son of *Gaurī* whose

place is reached by *yogīs*.

नमो वरदाय विघ्न हन्त्रे ॥१॥

We pray (*Gaṇeśa*) who is giver of boons and destroys obstacles.

नमस्ते शारदे देवि काश्मीरपुरवासिनि ।

त्वामहं प्रार्थये नित्यं विद्यादानं च देहि मे ॥३॥

(सरस्वतरहस्योपनिषद् २)

We pray *devī Śārada*, resident of *Kāśmīra* town. We request her to give knowledge daily.

यच्छून्यवादिनां शून्यं ब्रह्म ब्रह्मविदां च यत् ॥

विज्ञानमात्रं विज्ञानविदां यदमलात्मकम् ।

पुरुषः सांख्यदृष्टीनामीश्वरो योगवादिनाम् ॥

शिवः शैवागमस्थानां कालः कालैकवादिनाम् ।

यत्सर्वशास्त्रसिद्धान्तं यत्सर्व हृदयानुगम् ॥

यत्सर्वं सर्वगं वस्तु यत्तत्त्वं तदसौ स्थितः ।

(अन्नपूर्णोपनिषद् ३/१९-२२)

That Great being is zero among knowers of zero, *Brahma* among followers of this form, pure *vijñāna* (science) among scientists. He is *Puruṣa* as per *sāṅkhya*, *Īśvara* for *Yogīs*, *Śiva* for *śaivas*, *Kāla* (time) for measurers of time. He is all the principles of all texts and feelings in all hearts. Whatever object exists, He is within that.

References

1. The three books by *Svāmī Nīśchalānanda Sarasvatī* are- *Svastika Gaṇita*, *Aṅka-padīyam* (*Gaṇanātha-Gaṇita*), and *Gaṇita-darśana*. All these have been published by *Svasti Prakāśana Saṁsthāna*, *Govardhana Pīṭha*, *Purī*-752001, Orissa.
2. Original texts of *Vedas* have been published by *Chowkhamba Publishers*, *Jawahar Nagar*, *Delhi*-7 with hindi translation and *Sāyana* commentary. Collection of 108 *upaniṣads* also has been published by it. *Maitrāyaṇī Samhitā*, *Kāthaka saṁhitā* are by *Vedic Svādhyāya Maṇḍala Pardi*, *Valsad* (Gujrat). *Śatapatha Brāhmaṇa* with *Sāyana* commentary has been published by *Nag Publishers*, *Jawahar Nagar*, *Delhi*-7.
3. Astronomical measurements have been given in many books. Most common is Physics by Resnik and Haliday for plus 2, appendix.
4. Modern theories of numbers can be seen in any book on Real analysis. I have followed the treatment given by Topology by Simmons.
5. *Gītā* has been published with several commentaries by many publishers, e.g. Gita Press, Gorakhpur, UP.
6. Other references have been indicated in the text.